Topic 4E: Sequential Games and Stackelberg Oligopoly

Business Economics, Organization, and Management
(BUEC 211)

James Macek

Alberta School of Business
University of Alberta

Roadmap

1. **From Simultaneous to Sequential Games** — timing, game trees, and backward induction
2. **Quantity-Stackelberg Games** — first-mover advantage and why leading on output pays
3. **Price-Stackelberg Games** — second-mover advantage and price leadership
4. **Limit Pricing and Entry Deterrence** — using price as a strategic barrier to entry

From Simultaneous to Sequential Games

In Topic 4D, we studied **simultaneous** games: firms choose strategies at the same time without observing the rival's action.

From Simultaneous to Sequential Games

In Topic 4D, we studied **simultaneous** games: firms choose strategies at the same time without observing the rival's action.

Many real business decisions are **sequential**:

- One firm moves *first* and **commits** to a strategy.
- Rivals *observe* that choice, then respond.

From Simultaneous to Sequential Games

In Topic 4D, we studied **simultaneous** games: firms choose strategies at the same time without observing the rival's action.

Many real business decisions are **sequential**:

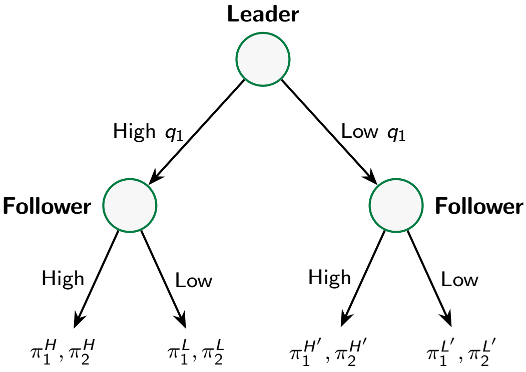
- One firm moves *first* and **commits** to a strategy.
- Rivals *observe* that choice, then respond.

Examples:

- Boeing announces a new aircraft programme; Airbus decides whether to match it.
- An incumbent airline expands route capacity; a low-cost carrier decides whether to enter the market.
- A dominant platform sets a low subscription price; a potential rival decides whether to launch.

The Extensive Form: Game Trees

Sequential games are represented by **game trees** (extensive form).



Reading the tree:

- **Nodes** — decision points (circles)
- **Branches** — available actions
- **Leaves** — payoffs ($\pi_{\text{Leader}}, \pi_{\text{Follower}}$)
- The Leader moves at the root; the Follower moves after observing the Leader's choice.

Backward Induction

Start at the **last** decision nodes and work *backward*. At each node, the player selects the action that maximises their own payoff, taking all future play as given.

Backward Induction

Start at the **last** decision nodes and work *backward*. At each node, the player selects the action that maximises their own payoff, taking all future play as given.

Why backward induction?

- The leader cannot simply ignore the follower — the follower will *react optimally* to whatever the leader does.
- The leader should **anticipate** that reaction and choose the output level that is best for itself, *given* the follower's optimal response.

Subgame Perfect Nash Equilibrium

The outcome of backward induction. Each player's strategy is optimal at *every* point in the game — even at nodes never reached in equilibrium.

Quantity-Stackelberg: Setup

Two firms compete on **quantity**. One firm — the **Leader** — chooses its output *first* and commits publicly. The other — the **Follower** — observes the leader's quantity and then chooses its own.

Quantity-Stackelberg: Setup

Two firms compete on **quantity**. One firm — the **Leader** — chooses its output *first* and commits publicly. The other — the **Follower** — observes the leader's quantity and then chooses its own.

Key departures from Cournot:

- **Cournot:** firms choose output *simultaneously*; neither observes the other's decision.
- **Stackelberg:** decisions are *sequential*; the follower's best-response function is therefore known and exploitable by the leader.

Quantity-Stackelberg: Setup

Two firms compete on **quantity**. One firm — the **Leader** — chooses its output *first* and commits publicly. The other — the **Follower** — observes the leader's quantity and then chooses its own.

Key departures from Cournot:

- **Cournot:** firms choose output *simultaneously*; neither observes the other's decision.
- **Stackelberg:** decisions are *sequential*; the follower's best-response function is therefore known and exploitable by the leader.

Sequence of play:

1. Leader chooses q_1.
2. Follower observes q_1, then chooses q_2.
3. Market price $P(q_1 + q_2)$ is determined; payoffs realised.

Example: American Airlines vs. United Airlines

We return to the airline duopoly: American Airlines (A) and United Airlines (U) from Topic 4D.

Market setup (same as Cournot):

$$P = 339 - Q, \qquad Q = q_A + q_U$$

$$MC_A = MC_U = \$147 \text{ per passenger}$$

New assumption — sequential timing:

- **American Airlines** is the **Stackelberg Leader**: it announces its route capacity q_A first (e.g., schedules new flights months in advance).
- **United Airlines** is the **Follower**: it observes American's capacity commitment and then sets q_U.

Step 1: The Follower's Problem (United Airlines)

United **takes** q_A **as given** and maximizes profit:

$$\pi_U = \big(P - 147\big)q_U = \big(339 - q_A - q_U - 147\big)q_U = \big(192 - q_A - q_U\big)q_U$$

Step 1: The Follower's Problem (United Airlines)

United **takes** q_A **as given** and maximizes profit:

$$\pi_U = \big(P - 147\big)q_U = \big(339 - q_A - q_U - 147\big)q_U = \big(192 - q_A - q_U\big)q_U$$

Setting $MR_U = MC_U$:

$$192 - q_A - 2q_U = 147 \quad \Longrightarrow \quad \boxed{q_U^*(q_A) = 96 - \tfrac{1}{2}\,q_A}$$

Step 1: The Follower's Problem (United Airlines)

United **takes** q_A **as given** and maximizes profit:

$$\pi_U = \big(P - 147\big)q_U = \big(339 - q_A - q_U - 147\big)q_U = \big(192 - q_A - q_U\big)q_U$$

Setting $MR_U = MC_U$:

$$192 - q_A - 2q_U = 147 \quad \Longrightarrow \quad \boxed{q_U^*(q_A) = 96 - \tfrac{1}{2}\,q_A}$$

United's best-response function is *identical* to its Cournot best response. The difference is that the leader **knows and uses** this function when it makes its decision.

Interpretation: every extra unit American produces leads United to reduce its output by **half** a unit. American can *push United down its best-response curve* by producing more and capturing more market share.

Step 2: The Leader's Problem (American Airlines)

American moves first. So it knows how United will respond to any decision it makes.

Step 2: The Leader's Problem (American Airlines)

American moves first. So it knows how United will respond to any decision it makes.

American substitutes United's best response function into its own profit function:

Step 2: The Leader's Problem (American Airlines)

American moves first. So it knows how United will respond to any decision it makes.

American substitutes United's best response function into its own profit function:

$q_U^* = 96 - \frac{1}{2}q_A$ into its own profit function:

$$\pi_A = \left(192 - q_A - q_U^*\right)q_A$$

Step 2: The Leader's Problem (American Airlines)

American moves first. So it knows how United will respond to any decision it makes. American substitutes United's best response function into its own profit function: $q_U^* = 96 - \frac{1}{2}q_A$ into its own profit function:

$$\pi_A = \left(192 - q_A - q_U^*\right) q_A$$

$$= \left(192 - q_A - \underbrace{\left(96 - \tfrac{1}{2}q_A\right)}_{q_U^*}\right) q_A = \left(96 - \tfrac{1}{2}q_A\right) q_A$$

$$\pi_A = 96\, q_A - \tfrac{1}{2}\, q_A^2$$

Step 2: The Leader's Problem (American Airlines)

American moves first. So it knows how United will respond to any decision it makes.
American substitutes United's best response function into its own profit function:
$q_U^* = 96 - \frac{1}{2}q_A$ into its own profit function:

$$\pi_A = \left(192 - q_A - q_U^*\right) q_A$$

$$= \left(192 - q_A - \underbrace{\left(96 - \tfrac{1}{2}q_A\right)}_{q_U^*}\right) q_A = \left(96 - \tfrac{1}{2}q_A\right) q_A$$

$$\pi_A = 96\, q_A - \frac{1}{2}\, q_A^2$$

American treats United's output as a *function of its own choice*, not as a fixed constant (as in Simultaneous Cournot).

Step 2: Solving the Leader's Problem

Maximise $\pi_A = 96\,q_A - \frac{1}{2}q_A^2$ over q_A:

$$\frac{d\pi_A}{dq_A} = 96 - q_A = 0 \quad \Longrightarrow \quad \boxed{q_A^* = 96}$$

Step 2: Solving the Leader's Problem

Maximise $\pi_A = 96\, q_A - \frac{1}{2} q_A^2$ over q_A:

$$\frac{d\pi_A}{dq_A} = 96 - q_A = 0 \quad \Longrightarrow \quad \boxed{q_A^* = 96}$$

Substitute back into United's best response:

$$q_U^* = 96 - \tfrac{1}{2}(96) = 96 - 48 = \mathbf{48}$$

Equilibrium outcomes:

$$Q^* = 96 + 48 = 144, \qquad P^* = 339 - 144 = \$195$$

$$\pi_A^* = (195 - 147) \times 96 = \$4{,}608\text{k}, \qquad \pi_U^* = (195 - 147) \times 48 = \$2{,}304\text{k}$$

Stackelberg vs. Cournot vs. Monopoly vs. Competition

	Monopoly	Cournot	Stackelberg	Competition
q_A	96	64	96	96
q_U	—	64	48	96
Total Q	96	128	144	192
Price P	\$243	\$211	\$195	\$147
π_A	\$9,216k	\$4,096k	\$4,608k	\$0
π_U	—	\$4,096k	\$2,304k	\$0

Stackelberg vs. Cournot vs. Monopoly vs. Competition

	Monopoly	Cournot	Stackelberg	Competition
q_A	96	64	96	96
q_U	—	64	48	96
Total Q	96	128	144	192
Price P	\$243	\$211	\$195	\$147
π_A	\$9,216k	\$4,096k	\$4,608k	\$0
π_U	—	\$4,096k	\$2,304k	\$0

- The **leader earns more** than in Cournot (**first mover advantage**)
- The **follower earns less** than in Cournot

  Consumers benefit: lower price and more output than Cournot.

Why Does First Mover Advantage Arise?

In the Cournot model, quantities are **strategic substitutes**: when one firm increases its output, the rival's best response is to *decrease* its output.

Why Does First Mover Advantage Arise?

In the Cournot model, quantities are **strategic substitutes**: when one firm increases its output, the rival's best response is to *decrease* its output.

This is the source of first mover advantage:

- By committing to a **large quantity**, the leader *shifts the game* in its favour — the follower is forced to produce less.
- The leader essentially "steals" market share from the follower.

Why Does First Mover Advantage Arise?

In the Cournot model, quantities are **strategic substitutes**: when one firm increases its output, the rival's best response is to *decrease* its output.

This is the source of first mover advantage:

- By committing to a **large quantity**, the leader *shifts the game* in its favour — the follower is forced to produce less.
- The leader essentially "steals" market share from the follower.

Real-world commitment devices:

- Building capacity in advance (airlines, oil refineries, steel plants).
- Long-term supply contracts.
- Public announcements of expansion plans.

Price Leadership: Does Order Matter?

New question: What if firms compete on *price*? Does being the first to announce a price give an advantage? **Case 1 — Homogeneous products:**

- If Firm 1 sets $p_1 > c$, Firm 2 can set $p_2 = p_1 - \varepsilon$ and capture the entire market.
- The leader is stuck: any $p_1 > c$ invites undercutting.
- Moving first in price (with homogeneous goods) is a disadvantage.

Case 2 — Differentiated products:

- Products are imperfect substitutes; a price cut does not steal *all* customers. But, the logic applies:
- The follower can *observe* the leader's price announcement and undercut.
- This creates a **second mover advantage**.

Price-Stackelberg: Setup (Spotify vs. Apple Music)

We return to the streaming duopoly from Topic 4D.

Demand (millions of subscribers per month):

$$q_S = 48 - 4p_S + 2p_A, \qquad q_A = 48 - 4p_A + 2p_S$$

Marginal cost: $c = \$6$ per subscriber.

Recall the simultaneous Bertrand equilibrium:

$$p_S^C = p_A^C = \$12, \qquad q_S^C = q_A^C = 24\text{M}, \qquad \pi_S^C = \pi_A^C = \$144\text{M}$$

New assumption — sequential timing:

- **Spotify** is the **price leader**: it announces p_S first (e.g., publicly announces its new subscription fee).

Step 1: The Follower's Best Response (Apple Music)

Apple takes p_S as given and maximises:

$$\pi_A = (p_A - 6)\, q_A = (p_A - 6)(48 - 4p_A + 2p_S)$$

Step 1: The Follower's Best Response (Apple Music)

Apple takes p_S as given and maximises:

$$\pi_A = (p_A - 6)\, q_A = (p_A - 6)(48 - 4p_A + 2p_S)$$

First-order condition:

$$\frac{d\pi_A}{dp_A} = (48 - 4p_A + 2p_S) + (p_A - 6)(-4) = 0$$

$$48 - 8p_A + 2p_S + 24 = 0 \quad \Longrightarrow \quad \boxed{p_A^*(p_S) = 9 + 0.25\, p_S}$$

Step 1: The Follower's Best Response (Apple Music)

Apple takes p_S as given and maximises:

$$\pi_A = (p_A - 6)\, q_A = (p_A - 6)(48 - 4p_A + 2p_S)$$

First-order condition:

$$\frac{d\pi_A}{dp_A} = (48 - 4p_A + 2p_S) + (p_A - 6)(-4) = 0$$

$$48 - 8p_A + 2p_S + 24 = 0 \quad \Longrightarrow \quad \boxed{p_A^*(p_S) = 9 + 0.25\, p_S}$$

Prices are **strategic complements**: if Spotify raises its price, Apple's best response is to *also raise* its price. This is the opposite of Cournot quantities (strategic substitutes).

Step 2: The Leader's Problem (Spotify)

Spotify substitutes Apple's best response $p_A^* = 9 + 0.25 p_S$ into its own demand:

$$q_S = 48 - 4p_S + 2(9 + 0.25p_S) = 48 - 4p_S + 18 + 0.5p_S = 66 - 3.5p_S$$

Step 2: The Leader's Problem (Spotify)

Spotify substitutes Apple's best response $p_A^* = 9 + 0.25p_S$ into its own demand:

$$q_S = 48 - 4p_S + 2(9 + 0.25p_S) = 48 - 4p_S + 18 + 0.5p_S = 66 - 3.5p_S$$

Spotify's profit:

$$\pi_S = (p_S - 6)(66 - 3.5p_S)$$

First-order condition:

$$\frac{d\pi_S}{dp_S} = (66 - 3.5p_S) + (p_S - 6)(-3.5) = 0$$

$$66 - 7p_S + 21 = 0 \quad \Longrightarrow \quad \boxed{p_S^* = \tfrac{87}{7} \approx \$12.43}$$

Apple's response:

$$p_A^* = 9 + 0.25(12.43) \approx \$12.11$$

Price-Stackelberg Equilibrium

Equilibrium quantities and profits:

$$q_S = 66 - 3.5(12.43) = 22.5\text{M subscribers}$$
$$q_A = 48 - 4(12.11) + 2(12.43) = 24.4\text{M subscribers}$$

$$\pi_S = (12.43 - 6) \times 22.5 \approx \$144.6\text{M}, \qquad \pi_A = (12.11 - 6) \times 24.4 \approx \$149.2\text{M}$$

Price-Stackelberg Equilibrium

Equilibrium quantities and profits:

$$q_S = 66 - 3.5(12.43) = 22.5\text{M subscribers}$$

$$q_A = 48 - 4(12.11) + 2(12.43) = 24.4\text{M subscribers}$$

$$\pi_S = (12.43 - 6) \times 22.5 \approx \$144.6\text{M}, \qquad \pi_A = (12.11 - 6) \times 24.4 \approx \$149.2\text{M}$$

	Simultaneous Bertrand	**Leader (Spotify)**	**Follower (Apple)**
Price	\$12.00	\$12.43	\$12.11
Quantity	24.0M	22.5M	24.4M
Profit	\$144.0M	\$144.6M	\$149.2M

The **follower (Apple) earns more** than either player under simultaneous pricing.

Why a Second Mover Advantage in Prices?

The logic:

- Spotify announces a *high* price first ($p_S = \$12.43 > \$12$).
- Because prices are **strategic complements**, Apple's best response is to also raise its price ($p_A = \$12.11 > \$12$).
- Apple captures more customers than Spotify (24.4M vs 22.5M) because it prices *slightly below* Spotify while still above cost.

Managerial implication: In price-setting industries, it often pays to *wait and see* what the rival charges before committing. Being the first to announce a price raise can be exploited by a rival who undercuts you slightly.

Remember this!

Model	**Strategic variable**	**First mover**
Cournot	Quantity (substitutes)	**Advantage**
Bertrand (diff.)	Price (complements)	**Disadvantage**

Limit Pricing and Entry Deterrence

A New Strategic Problem: Entry Deterrence

So far, we have assumed the number of firms is *fixed*.

A New Strategic Problem: Entry Deterrence

So far, we have assumed the number of firms is *fixed*.

Now consider: **one incumbent** facing a **potential entrant**.

- The **incumbent** currently enjoys monopoly profits.
- A **potential entrant** is deciding whether to pay a fixed entry cost F and enter the market.
- If the entrant enters, the market becomes a duopoly and profits fall for both.

A New Strategic Problem: Entry Deterrence

So far, we have assumed the number of firms is *fixed*.

Now consider: **one incumbent** facing a **potential entrant**.

- The **incumbent** currently enjoys monopoly profits.
- A **potential entrant** is deciding whether to pay a fixed entry cost F and enter the market.
- If the entrant enters, the market becomes a duopoly and profits fall for both.

The incumbent's problem: Can the incumbent use its *pricing strategy* to make entry unattractive — preserving its monopoly position?

A New Strategic Problem: Entry Deterrence

So far, we have assumed the number of firms is *fixed*.

Now consider: **one incumbent** facing a **potential entrant**.

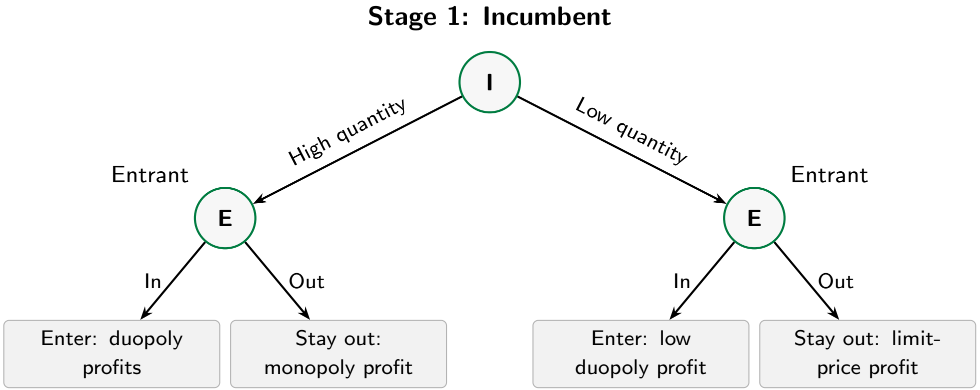
- The **incumbent** currently enjoys monopoly profits.
- A **potential entrant** is deciding whether to pay a fixed entry cost F and enter the market.
- If the entrant enters, the market becomes a duopoly and profits fall for both.

The incumbent's problem: Can the incumbent use its *pricing strategy* to make entry unattractive — preserving its monopoly position?

Definition: Entry Deterrence

Strategies an incumbent uses to make market entry unprofitable for a potential rival.

The Entry Game: Sequence of Play



The incumbent commits to production in Stage 1. The entrant decides in Stage 2. We solve by **backward induction**.

Numerical Example: Oil Refinery

Setup:

$$\begin{aligned} \text{Market demand:} \quad & P = 100 - Q \\ \text{Incumbent MC:} \quad & c_I = \$10 \\ \text{Potential entrant MC:} \quad & c_E = \$10 \\ \text{Fixed entry cost:} \quad & F = \$400 \end{aligned}$$

Incumbent's monopoly outcome (benchmark):

$$MR = 100 - 2Q = 10 \;\Rightarrow\; Q_m = 45, \quad P_m = \$55$$

$$\pi_m = (55 - 10) \times 45 = \$2{,}025$$

Numerical Example: Oil Refinery

Setup:

$$\begin{aligned} \text{Market demand:} \quad & P = 100 - Q \\ \text{Incumbent MC:} \quad & c_I = \$10 \\ \text{Potential entrant MC:} \quad & c_E = \$10 \\ \text{Fixed entry cost:} \quad & F = \$400 \end{aligned}$$

Incumbent's monopoly outcome (benchmark):

$$MR = 100 - 2Q = 10 \;\Rightarrow\; Q_m = 45, \quad P_m = \$55$$

$$\pi_m = (55 - 10) \times 45 = \$2{,}025$$

Q: Can the incumbent deter entry by producing *more* than the monopoly quantity?

Solving for the Limit Quantity

If the incumbent pre-commits to output Q_I, the entrant's optimal response

$$\pi_E = (100 - Q_I - q_E - 10)\, q_E - F = (90 - Q_I - q_E)\, q_E - F$$

Solving for the Limit Quantity

If the incumbent pre-commits to output Q_I, the entrant's optimal response

$$\pi_E = (100 - Q_I - q_E - 10)\, q_E - F = (90 - Q_I - q_E)\, q_E - F$$

Entrant's FOC:

$$90 - Q_I - 2q_E = 0 \;\Rightarrow\; q_E^*(Q_I) = \frac{90 - Q_I}{2}$$

Maximum entrant profit (before F):

$$\pi_E^{\max} = \left(\frac{90 - Q_I}{2}\right)^2 - F$$

Solving for the Limit Quantity

If the incumbent pre-commits to output Q_I, the entrant's optimal response

$$\pi_E = (100 - Q_I - q_E - 10)\, q_E - F = (90 - Q_I - q_E)\, q_E - F$$

Entrant's FOC:

$$90 - Q_I - 2q_E = 0 \;\Rightarrow\; q_E^*(Q_I) = \frac{90 - Q_I}{2}$$

Maximum entrant profit (before F):

$$\pi_E^{\max} = \left(\frac{90 - Q_I}{2}\right)^2 - F$$

Entry deterred when $\pi_E^{\max} \leq 0$:

$$\left(\frac{90 - Q_I}{2}\right)^2 \leq F = 400 \;\Rightarrow\; \frac{90 - Q_I}{2} \leq 20 \;\Rightarrow\; \boxed{Q_I \geq Q_L = 50}$$

Limit Price vs. Monopoly Price

	Monopoly	**Limit Pricing**
Output Q_I	45	50
Price P	\$55	\$50
Incumbent profit	\$2,025	\$2,000
Entry occurs?	Yes ($\pi_E = \$400 > F$)	No

Limit Price vs. Monopoly Price

	Monopoly	**Limit Pricing**
Output Q_I	45	50
Price P	\$55	\$50
Incumbent profit	\$2,025	\$2,000
Entry occurs?	Yes ($\pi_E = \$400 > F$)	No

Limit Price vs. Monopoly Price

	Monopoly	**Limit Pricing**
Output Q_I	45	50
Price P	\$55	\$50
Incumbent profit	\$2,025	\$2,000
Entry occurs?	Yes ($\pi_E = \$400 > F$)	No

Limit pricing costs the incumbent only $\$2{,}025 - \$2{,}000 = \$25$ in forgone profit *per period*. If keeping the rival out is worth more than \$25 per period, limit pricing is the optimal strategy.

Is Limit Pricing Credible?

After deterring entry, the incumbent has an incentive to raise prices back to monopoly levels P_m.

Is Limit Pricing Credible?

After deterring entry, the incumbent has an incentive to raise prices back to monopoly levels P_m. The entrant may *anticipate* this and ignore the low price — making limit pricing a non-credible threat.

Is Limit Pricing Credible?

After deterring entry, the incumbent has an incentive to raise prices back to monopoly levels P_m. The entrant may *anticipate* this and ignore the low price — making limit pricing a non-credible threat.

Commitment mechanisms that make low prices credible:

- **Excess capacity investment:** Build more capacity than needed for monopoly output Q_m. This makes it cheap to flood the market if entry occurs, making the threat of low post-entry prices credible.
- **Long-term supply contracts:** Lock in low prices with buyers before entry occurs.
- **Learning-by-doing:** High output now lowers future costs (e.g., semiconductors, solar panels), making sustained low prices rational.
- **Reputation:** In a repeated game, the incumbent builds a reputation for aggressive pricing that deters future potential entrants across many markets.

Limit Pricing in Practice

Predatory pricing is the real-world name for aggressive pricing designed to deter or eliminate rivals. It is subject to antitrust scrutiny.

Examples:

- **Pharmaceuticals:** Brand-name drug makers cut prices aggressively when generics threaten to enter the market, using bulk contracts with pharmacies to foreclose shelf space.
- **Airlines:** Incumbent carriers on a profitable route may increase frequencies and cut fares when a low-cost carrier announces entry (e.g., WestJet entering new Canadian markets).
- **Big Tech:** Dominant platforms price key services at zero (e.g., free email, maps) to deter rivals from building a user base in adjacent markets.

Limit Pricing in Practice

Predatory pricing is the real-world name for aggressive pricing designed to deter or eliminate rivals. It is subject to antitrust scrutiny.

Examples:

- **Pharmaceuticals:** Brand-name drug makers cut prices aggressively when generics threaten to enter the market, using bulk contracts with pharmacies to foreclose shelf space.
- **Airlines:** Incumbent carriers on a profitable route may increase frequencies and cut fares when a low-cost carrier announces entry (e.g., WestJet entering new Canadian markets).
- **Big Tech:** Dominant platforms price key services at zero (e.g., free email, maps) to deter rivals from building a user base in adjacent markets.

Key Takeaways for Sequential Games

Model	**Key result**
Quantity Stackelberg	**First mover advantage**
Price Stackelberg (differentiated)	**Second mover advantage**
Limit Pricing	**Entry deterrence**

Key Takeaways for Sequential Games

Model	**Key result**
Quantity Stackelberg	**First mover advantage**
Price Stackelberg (differentiated)	**Second mover advantage**
Limit Pricing	**Entry deterrence**

Note: Sequential games show that **timing and commitment matter**. Being first is advantageous when rivals respond by *backing off* (quantities), but disadvantageous when rivals respond by *matching or improving* (prices). Incumbents can also use sequential play to shape whether rivals enter a market at all. But these threats need to be **credible**.