Topic 4D: Oligopoly and Game Theory

Business Economics, Organization, and Management (BUEC 211)

James Macek

Alberta School of Business
University of Alberta

Roadmap

1. Measuring Competition — Concentration ratios and the HHI
2. Introduction to Game Theory — Strategic interaction and Nash equilibrium
3. The Prisoner's Dilemma and Cartels — Collusion and its fragility
4. Cournot Oligopoly — Quantity competition
5. Bertrand Oligopoly — Price competition with homogeneous products
6. Bertrand with Differentiated Products — Price competition when products differ

Measuring Competition

Oligopoly: A Different Market Structure

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We need new tools—starting with ways to *measure* how competitive a market is.

The Four-Firm Concentration Ratio (CR_4)

The n-firm concentration ratio is the sum of the market shares of the n largest firms:

$$CR_n = \sum_{i=1}^{n} s_i$$

where s_i is the market share (% of firm sales to industry sales) of the i-th largest firm.

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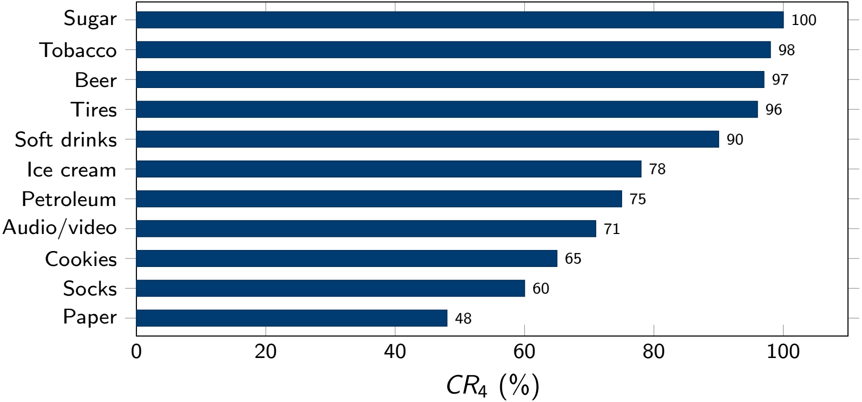
However, there are some limitations of CR_4:

- Excludes imports from foreign firms.
- Calculated for the national market—ignores local competition.
- Ignores competition between industries (e.g., bus vs. rail).
- Does not capture how shares are distributed *among* the top firms.

Concentration Ratios: Canadian Examples

Adapted from Parkin & Bade, Microeconomics: Canada in the Global Environment, 9th ed., 2015, Figure 12.1



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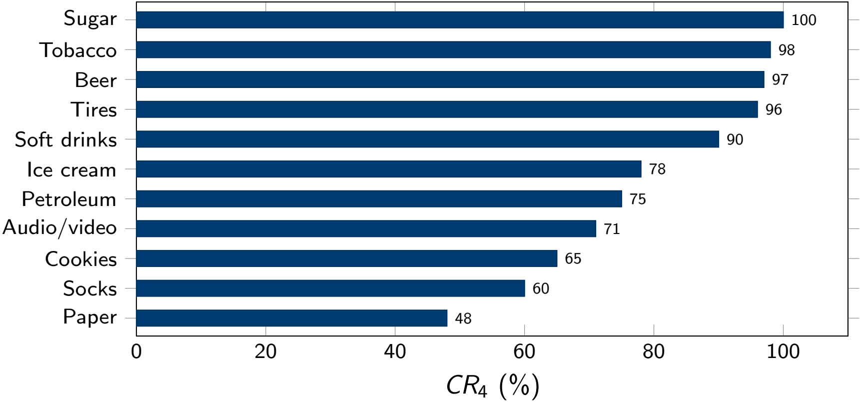


Industries with $CR_4 > 40\%$ are considered oligopolistic. All industries shown above exceed this threshold. Lower-concentration examples include pharmaceuticals (39%), printing (25%), and clothing (10%).

The Herfindahl-Hirschman Index (HHI)

The Herfindahl-Hirschman Index sums the *squared* market shares of **all** firms:

$$HHI = \sum_{i=1}^{N} s_i^2$$

where s_i is market share expressed as a percentage (so HHI ranges from near 0 to 10,000).

- **Pure monopoly:** one firm with 100% share $\Rightarrow$ $HHI = 10{,}000$ (its max value)
- **Duopoly** (50/50): $HHI = 50^2 + 50^2 = 5{,}000$
- **Many small firms:** $HHI \approx 0$ (perfect competition)

Squaring gives *disproportionate weight* to firms with large shares, capturing the inequality that CR_n misses.

HHI: A Worked Example

Suppose an industry has 5 firms with market shares: 40%, 25%, 15%, 12%, 8%.

$$HHI = 40^2 + 25^2 + 15^2 + 12^2 + 8^2 = 1600 + 625 + 225 + 144 + 64 = \mathbf{2{,}658}$$

Compare with equal shares

Five firms each with 20%: $HHI = 5 \times 20^2 = 2{,}000$.

The unequal industry is *more* concentrated even though $CR_5 = 100\%$ in both cases.

This is why regulators additionally consider the HHI: it captures both the *number* of firms and the *inequality* of their shares.

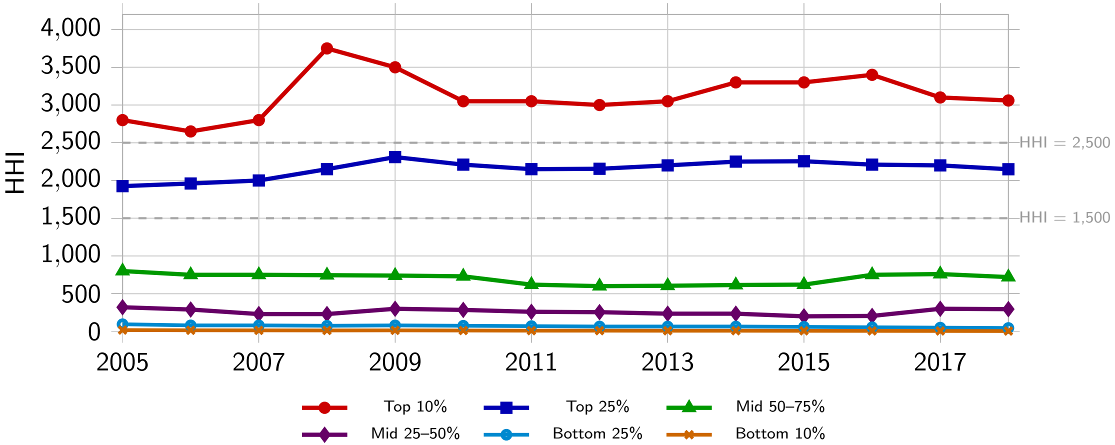
HHI Thresholds and Merger Review

Classification	HHI Range	Regulatory Concern
Unconcentrated	$< 1{,}500$	Low
Moderately Concentrated	1,500–2,500	Medium
Highly Concentrated	$> 2{,}500$	High

When the Competition Bureau assesses mergers, a key metric is ΔHHI: the change in HHI resulting from the merger.

- If firms with shares s_1 and s_2 merge: $\Delta HHI = 2\, s_1\, s_2$.
  If this is large, this can be used as evidence of an anti-competitive merger.

HHI Trends in Canada by Industry Concentration Tier, 2005–2018



Source: Competition Bureau Canada (2023), *Competition in Canada from 2000 to 2020: An Economy at a Crossroads*. Data from Statistics Canada, National Accounts Longitudinal Microdata File (NALMF), 4-digit NAICS industries.

Modeling Oligopolies

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If so, why do we need a special model of oligopoly?

- In competitive markets, firms are not strategic. (They produce such that $P = MC$; competitors' decisions don't matter.)

- Monopolists are not strategic because there are no competitors to worry about.

- This is not true for oligopolies. Oligopolists are large relative to the market, and the actions of one oligopolist make large differences in the profits of another.

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- This is not true for oligopolies. Oligopolists are large relative to the market, and the actions of one oligopolist make large differences in the profits of another.
- Oligopolies are best analyzed using a specialized field of study called **game theory**.

Game Theory and the Prisoner’s Dilemma

Why Game Theory?

In oligopoly, each firm's profit depends on what *other firms* do. This is strategic interdependence.

Definition: Game Theory

The study of decision-making in situations where attaining one's goals depends on interactions with others.

Game theory was developed in the 1940s and has become the primary tool for analyzing oligopoly. Every "game" has three components:

1. Players — the decision-makers (firms, countries, individuals)
2. Strategies — the actions available to each player
3. Payoffs — the outcome each player receives for every combination of strategies

Simultaneous Games and the Payoff Matrix

In a simultaneous game, players choose strategies at the same time (or without observing the other's choice).

Player 2

Player 1		Left	Right
	Up	a_1 , a_2	b_1 , b_2
	Down	c_1 , c_2	d_1 , d_2

Player 1's payoff

Player 2's payoff

Each cell shows payoffs for every combination of strategies.

Dominant Strategies and Nash Equilibrium

Dominant Strategy

A strategy that gives a player a higher payoff *regardless* of what the other player does.

A **rational player** always plays a dominant strategy if one exists.

Nash Equilibrium

A set of strategies—one for each player—such that no player can improve their payoff by *unilaterally* changing their strategy.

- Every dominant-strategy equilibrium is a Nash equilibrium, but not vice versa.
- Named after mathematician John Nash (1928–2015).

The Prisoner's Dilemma: Setup

Two suspects are arrested and interrogated in **separate cells**. Each can either Stay Silent (cooperate with each other) or Confess (defect).

- Both silent: each gets **1 year** (minor charge only).
- One confesses, the other silent: confessor goes **free**; silent one gets **10 years**.
- Both confess: each gets **5 years**.

Each suspect wants to minimize their own prison time. What should they do?

The Prisoner's Dilemma: Payoff Matrix

		Suspect B	
		Silent	Confess
Suspect A	Silent	-1 , -1	-10 , 0
	Confess	0 , -10	-5 , -5

Payoffs are in years of prison (more negative = worse).

The Prisoner's Dilemma: Payoff Matrix

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Suspect A		Silent	Confess
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	Confess	0 , -10	$\boxed{-5\ ,\ -5}$

Nash Equilibrium

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The Prisoner's Dilemma: Analysis

Suspect A's reasoning:

- If B stays silent: Confess $(0) >$ Silent (-1) $\Rightarrow$ Confess
- If B confesses: Confess $(-5) >$ Silent (-10) $\Rightarrow$ Confess

Confess is a dominant strategy for A. It is also dominant for B.

The Dilemma

Both suspects confess, receiving 5 years each. But if both had stayed silent, they would have served only 1 year each!

Individual rationality leads to a collectively worse outcome.

A Business Prisoner's Dilemma: Spotify vs. Apple

The prisoner's dillema game applies to oligopolistic markets. For example, Spotify and Apple compete on music streaming subscription pricing:

		Apple	
		\$14.99	\$9.99
Spotify	\$14.99	\$10M , \$10M	\$5M , \$15M
	\$9.99	\$15M , \$5M	$\boxed{\$7.5M , \$7.5M}$

Nash Equilibrium

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Apple

Spotify	Apple: $14.99	Apple: $9.99
$14.99	$10M , $10M	$5M , $15M
$9.99	$15M , $5M	$7.5M , $7.5M

Nash Equilibrium

Both charge $9.99 (dominant strategy for each). But both would prefer ($14.99, $14.99)!

Analyzing the Spotify–Apple Game

Spotify's reasoning:

- If Apple charges \$14.99: charge \$9.99 (\$15M $>$ \$10M). ✓
- If Apple charges \$9.99: charge \$9.99 (\$7.5M $>$ \$5M). ✓

Apple's reasoning is symmetric: \$9.99 is dominant for Apple too.

The Dilemma in Business

Each firm earns \$7.5M at the Nash equilibrium, but could earn \$10M if they cooperated. The \$2.5M gap is the "cost of competition" – competition brings down prices.

This is why firms are tempted to **collude** and behave jointly as a monopolist. This is also why economists worry about the formation of **cartels**.

Cartels: Collusion and Its Fragility

From the Prisoner's Dilemma to Cartels

The Spotify–Apple example illustrates a general insight:

Definition: The Oligopolist's Dilemma

Firms would earn higher profits if they could **cooperate** (e.g., both charge high prices). But each firm has an individual incentive to **defect** (undercut the rival).

This is exactly the logic behind cartels: groups of firms that collude to restrict output and raise prices, approximating a monopoly outcome and sharing the rents.

Most famous example: OPEC—member countries agree to limit oil production to keep prices high.

How Cartels Work

1. **Agreement:** Member firms agree to reduce output below their individually profit-maximizing levels.
2. **Goal:** Restrict total market supply to (ideally) the monopoly level, raising the market price.
3. **Division of rents:** Cartel members divide the monopoly profits among themselves—typically by assigning production quotas.

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If it works, each firm earns more than it would under oligopoly competition. The cartel collectively acts as a monopolist.

Why Cartels Collapse: The Cheating Incentive

		Firm 2: Honor Quota (Cartel Outcome)	Firm 2: Cheat
Firm 1	Honor Quota	\$50M , \$50M	\$20M , \$60M
	Cheat	\$60M , \$20M	\$35M , \$35M (Nash Equilibrium)

Cheating (producing more oil than the OPEC quota) is a dominant strategy: the cartel agreement unravels through *exactly* the Prisoner's Dilemma logic.

External Factors That Undermine Cartels

1. **Legal prohibition:** Cartels are illegal in Canada, the U.S., and the EU. The risk of heavy fines and jail time deters collusion.

2. **Insufficient market control:** If the cartel does not control enough of the market, non-member firms expand output and undercut the cartel price.

3. **Entry:** High cartel prices attract new entrants (unless barriers to entry are very high), eroding the cartel's market share over time.

Internal Factors That Undermine Cartels

Even without legal enforcement, cartels face internal pressures:

- **Incentive to cheat:** Each member can increase profit by secretly producing above its quota while others stick to theirs.
- **Difficulty of detection:** Cheating may be hard to observe, especially with many members or complex products.
- **Coordination costs:** More members $\Rightarrow$ harder to negotiate and enforce agreements.
- **Asymmetric firms:** Firms with different costs or capacities disagree about optimal quotas and price levels.

How Cartels Try to Sustain Themselves

To survive, a cartel needs mechanisms to **detect cheating** and **punish violators**:

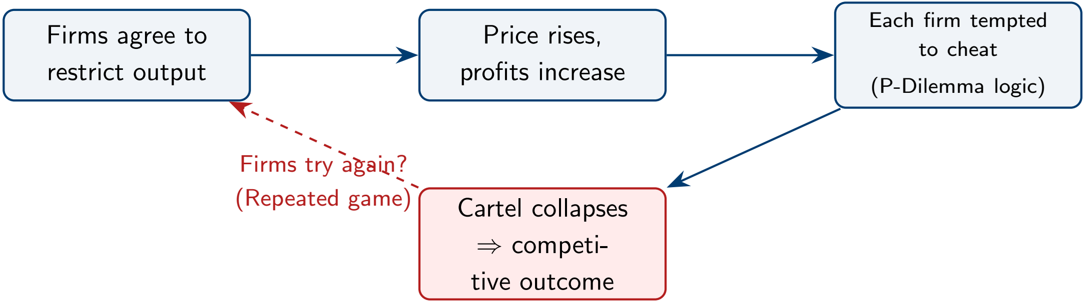
Detection mechanisms:

- Inspect each other's accounts or financial records.
- Divide the market by region or customer type.
- Use industry organizations to collect and monitor market-share data.

Enforcement mechanisms:

- Most-favored-customer clauses: Firms commit not to offer a lower price to *any* buyer without extending the same price to all contract holders—makes secret discounting visible.
- Threat of price wars: "If you cheat, we all flood the market."

Cartels: Summary



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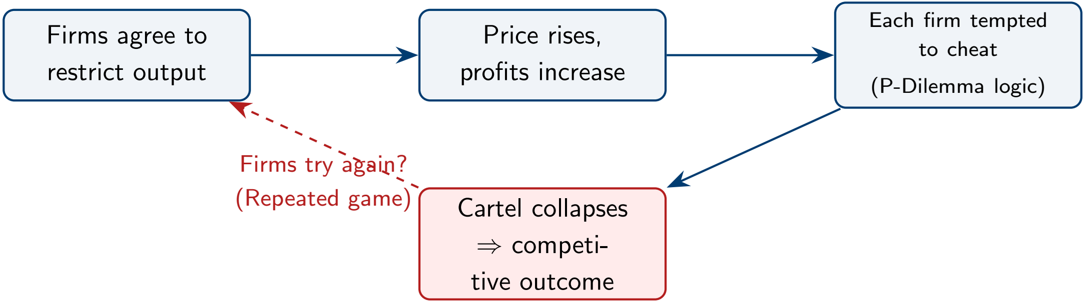

The Prisoner's Dilemma predicts cartel collapse in a *one-shot* game. However, repeated interactions may help sustain collusion through **punishment** strategies.

Oligopoly

Types of Oligopoly Models

If firms are *not* colluding, they compete independently. The appropriate model depends on the **strategic variable**:

Model	Strategic Variable	Best For
Cournot	Quantity	Industries where output (capacity) is chosen in advance, e.g. oil, airlines
Bertrand	Price	Industries where prices are chosen and capacity constraints are less important (e.g. Apple Music and Spotify memberships)

We start with the Cournot model, then turn to Bertrand.

Cournot Model: Assumptions

The Cournot model rests on four key assumptions:

1. **Few firms and no entry.**
2. **Identical (or similar enough) costs.**
3. **Identical (homogeneous) products.**
4. **Firms choose output levels independently and simultaneously.**

Firms set quantities; the *market price* adjusts until the market clears. Because each firm's output choice affects the market price, profits are interdependent.

Cournot Duopoly: Setup

Example: American Airlines (A) and United Airlines (U) are the only two carriers on a route.

Inverse demand function:

$$P = 339 - Q \quad \text{where} \quad Q = \underbrace{q_A + q_U}_{\text{Flights from } A \text{ and } U \text{ are perfect substitutes}}$$

Both airlines have constant marginal (and average) cost:

$$MC = AC = \$147 \text{ per passenger}$$

Key idea: Each airline takes the other's quantity as *given* and chooses its own output to maximize profit.

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Therefore, equilibrium of this model is a **Nash equilibrium**!

Residual Demand and Best Response

American's residual demand (taking q_U as given):

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To profit maximize, American sets $MR_A = MC$:

$$(339 - q_U) - 2q_A = 147 \quad \Rightarrow \quad q_A^* = \frac{192 - q_U}{2} = 96 - \frac{q_U}{2}$$

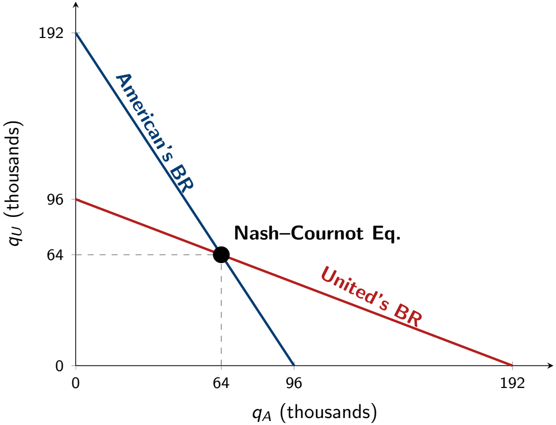
$q_A^\star$ is American's best-response function – how many flights American should provide to maximize profits given q_U flights provided by United.

Residual Demand and Best Responses

These airlines produce perfect substitutes at the same cost, so these firms are identical. This means United's best response function is the same as American's:

$$q_U^* = 96 - \frac{q_A}{2}$$

Best-Response Curves and a Nash equilibrium



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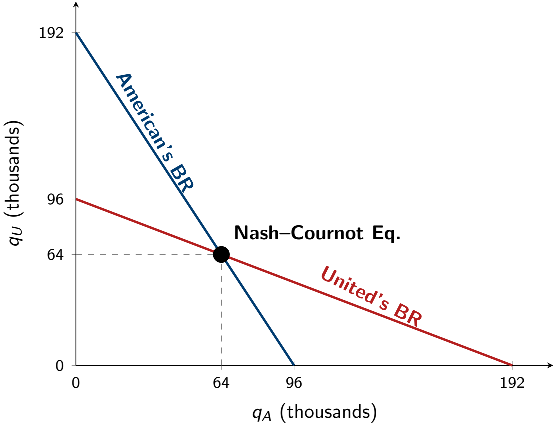

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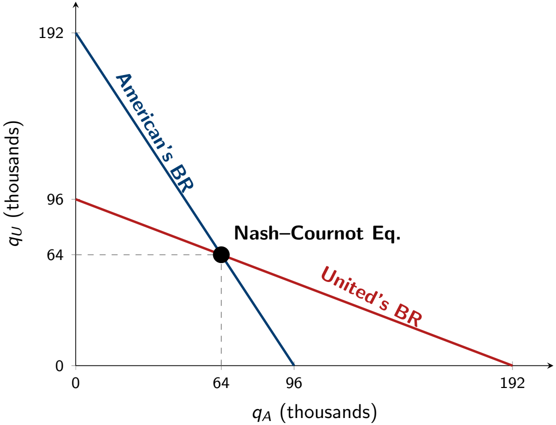
Best-Response Curves and a Nash equilibrium



A Nash equilibrium occurs at the intersection of the best response curves. Each firm is choosing capacity to maximize profits taken as given the capacity of the other airline.

Solving for the Nash–Cournot Equilibrium

Substitute United's best response into American's:

$$q_A = 96 - \frac{1}{2}\left(96 - \frac{q_A}{2}\right) = 96 - 48 + \frac{q_A}{4} = 48 + \frac{q_A}{4}$$

$$\frac{3}{4} q_A = 48 \quad \Rightarrow \quad q_A^* = 64$$

Because Airline and United are identical firms, it is easy to show that $q_U^* = 64$, as well.

Equilibrium outcomes:

$$Q^* = 64 + 64 = 128$$

$$P^* = 339 - 128 = \$211$$

$$\pi_A = \pi_U = (211 - 147) \times 64 = \$4{,}096 \text{ (thousands)}$$

Cournot Oligopoly vs. Monopoly vs. Competition

	Monopoly	**Cournot Duopoly**	**Competition**
Total Q	96	128	192
Price P	\$243	\$211	\$147
Industry Profit	\$9,216	\$8,192	\$0

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Total Q	96	128	192
Price P	\$243	\$211	\$147
Industry Profit	\$9,216	\$8,192	\$0

The Cournot oligopoly outcome lies **between** monopoly and perfect competition:

- More output and lower price than monopoly.
- Less output and higher price than perfect competition.

Cournot Oligopoly with a large number of firms

With n identical firms, inverse demand $P = a - Q$, and constant $MC = c$:

$$q_i^* = \frac{a-c}{n+1}, \qquad Q^* = \frac{n(a-c)}{n+1}, \qquad P^* = \frac{a+n\,c}{n+1}$$

As $n \to \infty$:

- $P^* \to c$ (the competitive price $= MC$)

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As $n \to \infty$:

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- $Q^* \to a - c$ (the competitive quantity)

$\Longrightarrow$ Cournot competition converges to perfect competition as the number of firms gets large!

What if one firm provides flights at lower cost?

Suppose United's marginal cost falls from \$147 to \$99, while American's stays at \$147.

New best responses:

$$q_A^* = 96 - \frac{q_U}{2} \quad \text{(unchanged)}$$

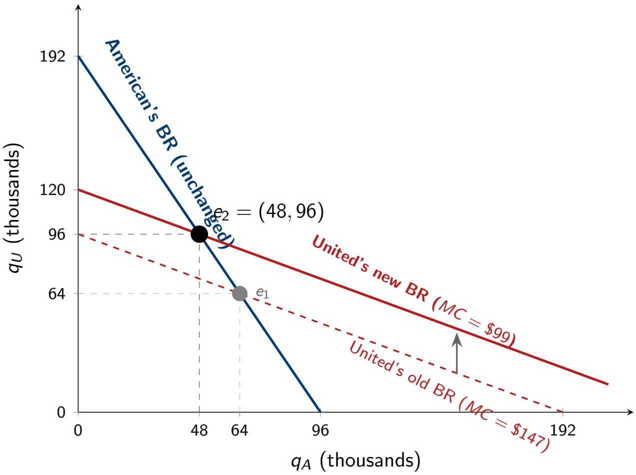
$$q_U^* = \frac{339 - 99}{2} - \frac{q_A}{2} = 120 - \frac{q_A}{2} \quad \text{(shifted out)}$$

New equilibrium: Solve simultaneously:

$$q_A = 96 - \tfrac{1}{2}(120 - q_A/2) = 36 + q_A/4 \;\Rightarrow\; q_A^* = 48$$
$$q_U^* = 120 - 24 = 96$$

$\Longrightarrow$ The low-cost firm produces *more*; the high-cost firm produces *less*. Total output rises and price falls.

Asymmetric Costs: Shift in Best-Response Curves



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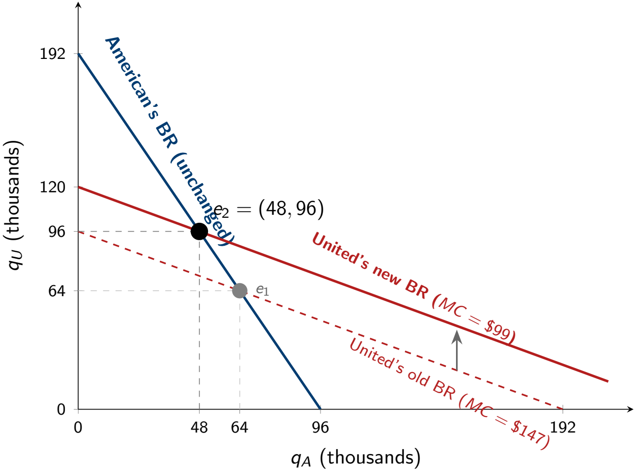

When United's *MC* falls, its BR shifts *outward*: more flights are produced for every choice of flight capacity by American.

Taxes in the Cournot Model

- In the Cournot model (and the monopoly model), imposing a tax on consumption implies **incomplete pass through** of the tax on prices, even with constant marginal costs of production.
- For example, imposing a $1 per litre tax on oil paid by consumers will not raise prices paid by consumers by $1, it will be less than $1.

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- The reason why is due to market power. Imposing a tax and increasing prices may make demand more elastic, which means monopolists/oligopolists charge a lower markup over marginal cost.
- In the Cournot model we discussed with a large number of firms, there will be almost **complete pass through**.

Bertrand Oligopoly (Homogeneous Products)

Bertrand Competition: Overview

In the Bertrand model, firms compete on **price** rather than quantity. This turns out to have very different implications relative to the Cournot model.

Key assumptions (homogeneous products):

1. Two (or more) firms produce **identical** products (or perfect substitutes)
2. Firms set prices **simultaneously and independently**.
3. Consumers buy from the firm with the lowest price. If prices are equal, consumers split evenly.
4. Each firm can supply the entire market **(no capacity constraints)**
5. Constant marginal costs

Key question: What prices will the firms choose?

Bertrand Logic: Undercutting

Consider any price $p_1 > c$:

- If $p_2 > p_1$: Firm 1 captures the *entire* market.
- If $p_2 = p_1$: Firms split the market.
- If $p_2 < p_1$: Firm 1 gets *zero* sales.

Incentive to undercut: Whenever $p_1 > c$, Firm 2 can profitably set $p_2 = p_1 - \$0.01$ and steal all of Firm 1's customers. And vice versa.

This undercutting continues until...

The Bertrand Paradox

The unique Nash–Bertrand equilibrium with homogeneous products is:

$$\boxed{p_1^* = p_2^* = c}$$

Both firms price at marginal cost. With constant marginal costs, this means that each firm's profits are zero.

The Bertrand Paradox

With just **two** firms and identical products, price competition drives the outcome all the way to the **perfectly competitive** result.

Why "paradox"? It seems implausible that only two firms would compete so aggressively so as to drive price down to marginal cost, and each firm earns zero profit.

Why Is $p = c$ a Nash Equilibrium?

Check: Can either firm profitably deviate from $p_1 = p_2 = c$?

- **Raise price above c?** You lose all customers to the rival who stays at c. Profit $= 0$. No improvement.
- **Lower price below c?** You capture the whole market...at a loss on every unit. Profit < 0. Strictly worse.
- **Stay at c?** Profit $= 0$, but no profitable deviation exists.

Neither firm can improve by unilaterally changing its price $\Rightarrow$ Nash equilibrium. ✓

Cournot vs. Bertrand: Comparison

	Cournot	Bertrand (Homogeneous)
Strategic variable	Quantity	Price
Equilibrium price	$P > MC$	$P = MC$
Equilibrium profit	Positive	Zero
Converges to competition	As $n \to \infty$	With just $n = 2$
Best suited for	Capacity-constrained	Price-setting industries

Bertrand with Differentiated Products

Why Differentiation Matters

The Bertrand paradox arises because products are *perfect substitutes*: a tiny price cut steals the entire market.

In reality, most products are differentiated:

- iPhone vs. Samsung Galaxy — different ecosystems, design, features.
- Tim Hortons vs. Starbucks — different branding, menu, ambiance.

With differentiation, a firm that raises its price loses *some* (but not all) customers. This breaks the "Bertrand paradox" and allows firms to earn positive profits.

Differentiated Bertrand: Demand for each product

With two firms selling *differentiated* products, each firm faces its own downward-sloping demand that depends on *both* prices:

$$q_1 = a - b\,p_1 + d\,p_2$$
$$q_2 = a - b\,p_2 + d\,p_1$$

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Interpretation of parameters:

- $b > 0$: own-price effect (raising p_1 reduces q_1).
- $d > 0$: cross-price effect (raising p_2 *increases* q_1—consumers switch).
- $b > d$: own-price effect is greater than cross-price (standard)
- $a > 0$: base demand when all prices are zero.

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When $d = 0$, each firm is **effectively a monopolist**. Why?

Example: Spotify vs. Apple Music

Spotify and Apple Music sell *differentiated* streaming subscriptions. Some users prefer Spotify's playlists and social features; others prefer integration with other Apple products.

Demand (millions of subscribers per month):

$$q_S = 48 - 4\,p_S + 2\,p_A$$
$$q_A = 48 - 4\,p_A + 2\,p_S$$

Both platforms have $MC = \$6$ per subscriber per month (content licensing, royalties, and infrastructure).

Here $a = 48$, $b = 4$, $d = 2$, $c = 6$.

Spotify's Profit-Maximization Problem

Spotify chooses p_S to maximize profit taking p_A as given:

$$\pi_S = (p_S - 6)\, q_S = (p_S - 6)(48 - 4\, p_S + 2\, p_A)$$

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$$48 - 8\, p_S + 2\, p_A + 24 = 0$$

Solving for p_S:

$$\boxed{p_S^*(p_A) = \frac{72 + 2\, p_A}{8} = 9 + 0.25\, p_A}$$

Best-Response Functions

Spotify's best response:

$$p_S^*(p_A) = 9 + 0.25\, p_A$$

Because we assume Apple has the same demand and costs that Spotify has, Apple Music's best response is also:

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2. **Slope** < 1: Spotify responds *less* than one-for-one to Apple's price increase $(0.25 < 1)$.
3. **Intercept** $> MC$: Even if Apple priced at cost (\$6), Spotify would charge $9 + 0.25(6) = \$10.50 > \$6$. Differentiation protects Spotify's margin.

Solving for the Nash–Bertrand Equilibrium

At equilibrium, both best responses hold simultaneously.

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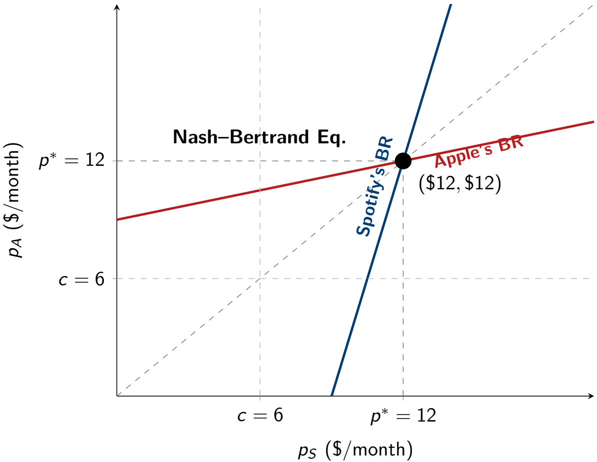
Equilibrium outcomes:

$$q_S = q_A = 48 - 4(12) + 2(12) = 48 - 48 + 24 = \mathbf{24} \text{ million subscribers}$$

$$\pi_S = \pi_A = (12 - 6) \times 24 = \$\mathbf{144} \text{ million per month}$$

Graphical Equilibrium: Spotify vs. Apple Music



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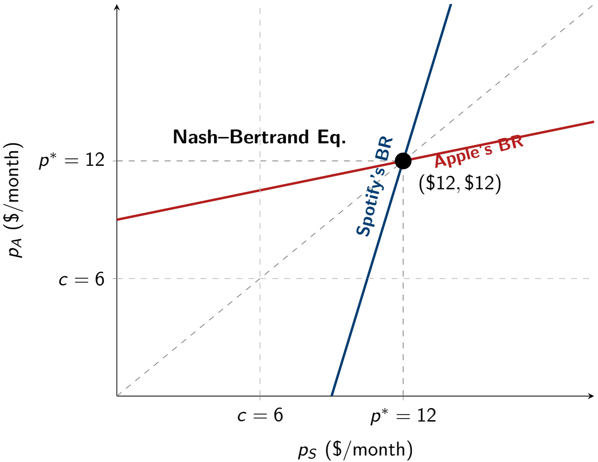

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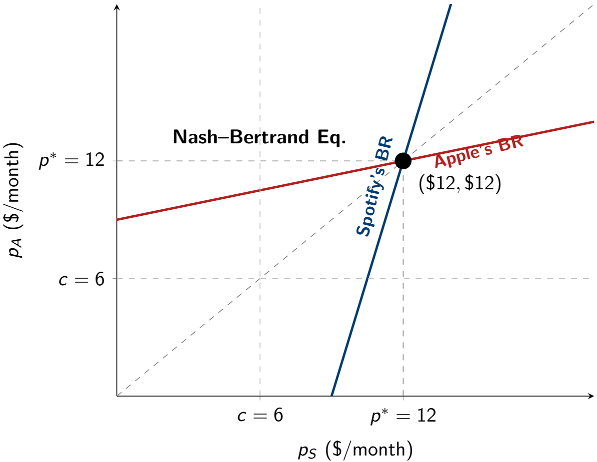

Notice: Best Response curves are **upward sloping**. Exercise: contrast that with the Cournot Model's BR curves. How does Apple react if Spotify's costs fall?

Interpreting the Equilibrium

Firm statistics	**Value**
Price per month	\$12.00
Marginal cost	\$6.00
Subscribers (each firm)	24 million
Monthly profit (each)	\$144 million

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Compare with the homogeneous Bertrand result:

- If Spotify and Apple Music were *perfect substitutes*, the equilibrium would be $p = MC = \$6$, with **zero** profit for both.
- Product differentiation allows a \$6 markup per subscriber—even with only two competitors.

What If Apple Unilaterally Cuts Its Price?

Suppose Apple deviates to $p_A = \$10$ while Spotify stays at $p_S = \$12$:

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Apple's outcome:

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Why?

Differentiation means Apple doesn't steal *all* of Spotify's customers. The demand gain from undercutting (+8M subs) is not large enough to compensate for the margin loss on the original 24M subscribers. This is why $p^* = \$12$ is an equilibrium.

Comparing All Oligopoly Models So Far

	Cournot	Bertrand (Homogeneous)	Bertrand (Differentiated)
Strategic variable	Quantity	Price	Price
Products	Identical	Identical	Differentiated
Eq. price	$MC < P < P^M$	$P = MC$	$MC < P < P^M$
Eq. profit	$\pi > 0$	$\pi = 0$	$\pi > 0$
Key driver	Output restraint	Undercutting	Product loyalty
$\#$ firms for $P = MC$	$n \to \infty$	$n = 2$	Depends ...

Key Takeaways

1. Measuring competition: CR_4 and HHI quantify market concentration. HHI is preferred because it captures inequality among firms.
2. Game theory: Oligopolists face strategic interdependence. Nash equilibrium is the central solution concept.
3. Prisoner's Dilemma and cartels: Firms would profit from collusion, but each has a dominant strategy to cheat. Cartels are inherently fragile.
4. Cournot: Quantity competition produces outcomes between monopoly and perfect competition, converging to the competitive outcome as $n \to \infty$.
5. Bertrand (homogeneous): Price competition with identical products drives price to MC—the Bertrand paradox.
6. Bertrand (differentiated): Product differentiation resolves the paradox—firms earn positive profits, with markups depending on d/b.