Topic 3B: Cost Minimization and Cost Functions

Business Economics, Organization, and Management
(BUEC 211)

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Outline

- Conceptualizing costs – how do we account for different forms of economic cost that affect firm decision making?
- Quantifying common measures of costs
- Costs in both the **short** run and **long** run

Conceptualizing Costs

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- But the technical relationship between inputs and outputs defined by the production process is only one determinant of how firms choose what to produce.
- We also need to account for the **cost of inputs**.

Explicit and Implicit Costs

- Financial accounting statements correctly measure costs for tax purposes and to meet other legal requirements.
- To make sound managerial decisions, good managers need more information.
- We need to think about both **explicit** and **implicit** costs.
- **Explicit costs** are direct, out-of-pocket payments for labor, capital, energy and materials.
- **Implicit costs** reflect forgone opportunities rather than explicit or current expenditure.

Opportunity Costs

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Example (Opportunity Costs)

Taylor owns and manages a consulting firm. The firm's monthly revenue is \$55k, and monthly costs are \$46k, which includes a \$2k per month salary for Taylor. Taylor also collects the firm's profits each month. Alternatively, Taylor could work for another firm and receive \$12k per month. Given this, is Taylor using her labor effectively? Why or why not?

Trumpian Opportunity Costs

https://www.youtube.com/watch?v=p5NYlRd15hU

Opportunity Costs

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- It is more difficult to determine the opportunity costs of inputs that are **durable**.

- Durable inputs are useable for a long period, perhaps for many years.

  e.g Capital, such as land, buildings, and equipment.

Durable Input Costs

- Two issues with durable inputs:
  1. How do we allocate the initial purchase cost over time?
  2. What do we do if the value of the input changes over time?
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Q: What are these costs? 1) The interest rate (the opportunity cost from selling the input and investing the proceeds in a bond), and 2) maintenance and depreciation expenses

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2. Next, suppose that there is no rental market for trucks. Should Tony operate the truck? No, because the user cost is likely \$2.25k/month.

Sunk Costs

- Some costs that appear in financial accounts are not relevant for managerial decisions.
- These are **sunk costs**: past expenditures that cannot be recovered.
- Sunk costs should be **ignored** by managers when making decisions.
- If an expenditure is sunk, it is **not** an opportunity cost.

Example (Sunk Costs)

A.T. & Love is a real estate developer that invested \$10 million to purchase an apartment building with the intent to renovate each unit into luxury condominiums. The renovation is expected to cost an additional \$2.5 million. However, since the initial purchase, a recession has hit, meaning demand for luxury condos has dried up. Gerald, one of A.T. & Love's executives, suggests the company invest in the renovations anyway, given that "We've already spent \$10 million on this thing—we might as well spend the additional \$2.5 to finish it up." Do you agree with Gerald's reasoning? Why or why not?

Common Measures of Cost

Key Cost Measures

Three key cost measures based on opportunity cost of inputs:

- **Fixed Cost (F)**: A cost that does not vary with the level of output.

  Includes expenditures on land, office space, and other overhead.

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- **Variable Cost (VC)**: A cost that changes as output changes.

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- **Total Cost (C)**: The sum of fixed and variable costs.

$$C = F + VC$$

Average Costs

Corresponding average cost measures:

- **Average Fixed Cost (AFC)**: $AFC = \dfrac{F}{q}$

  Falls as output rises because the fixed cost is spread over more units.

  Average Variable Cost (AVC): $AVC = \dfrac{VC}{q}$

  Can rise or fall with increases in output. (Why?)

- **Average Cost (AC)**: $AC = AFC + AVC$

  Also referred to as average total cost.

  May increase or decrease with increases in output. (Why?)

Marginal Cost

Another important cost concept: **Marginal Cost**

- Marginal Costs (MC) are the amount by which a firm's total cost changes if the firm produces one more unit.
$$MC = \frac{\Delta C}{\Delta q}$$
- Marginal costs are also equal to the change in variable costs from a one-unit increase in output:
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Q: Why does $MC = \frac{\Delta VC}{\Delta q}$? Fixed costs don't affect the rate at which production costs respond to increases in output.

Cost Example: Axel's Coffee

- As an example, consider Axel's coffee shop again.
- Axel is monitoring how much it costs him to produce different amounts of espresso per day.
- Axel is interested in understanding how his costs change as he sells more espresso.

Cost Example: Axel's Coffee

q	F	VC	C	MC	$AFC = \frac{F}{q}$	$AVC = \frac{VC}{q}$	$AC = \frac{C}{q}$
0	5	0.00	5.00				
1	5	3.01	8.01	3.01	5.00	3.01	8.01
2	5	5.00	10.00	1.99	2.50	2.50	5.00
3	5	6.73	11.73	1.73	1.67	2.24	3.91
4	5	8.37	13.37	1.65	1.25	2.09	3.34
5	5	10.00	15.00	1.63	1.00	2.00	3.00
6	5	11.73	16.73	1.73	0.83	1.96	2.79
7	5	13.75	18.75	2.02	0.71	1.96	2.68
8	5	16.85	21.85	3.10	0.63	2.11	2.73

Note: Output is in tens of litres per day, costs are in hundreds.

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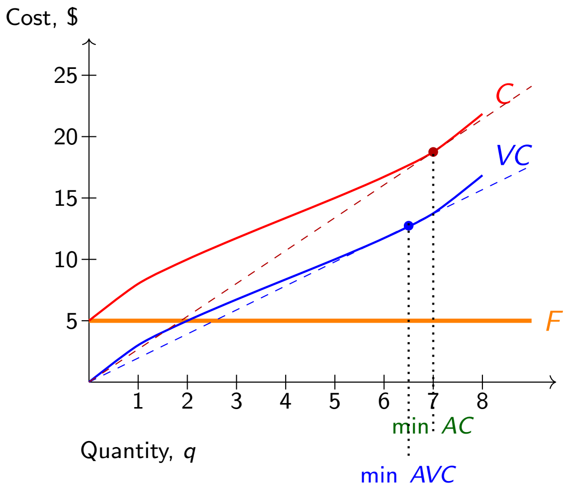
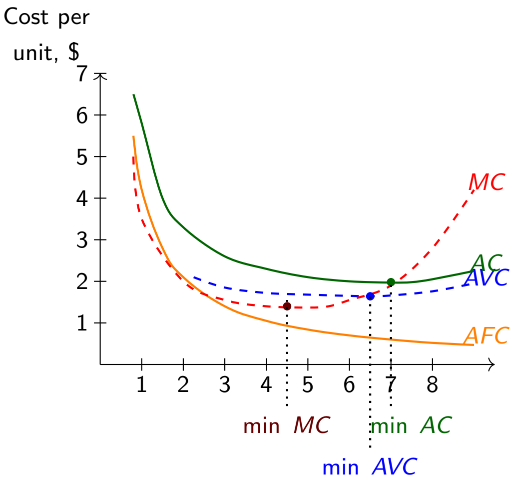
Note the cells in red. When marginal costs exceed average costs, average costs are increasing. (Sound familiar?)

Plotting Axel's Cost Curves





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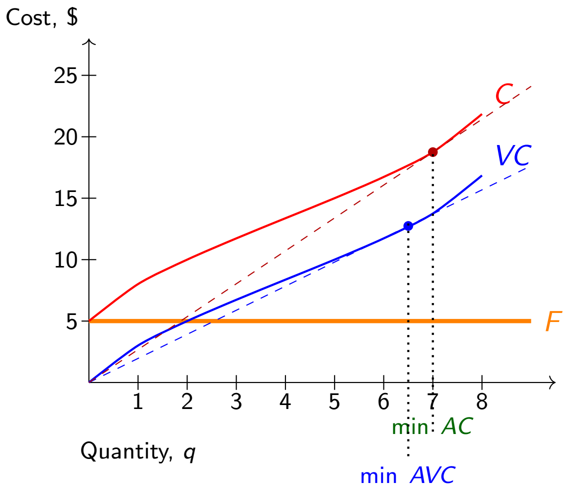

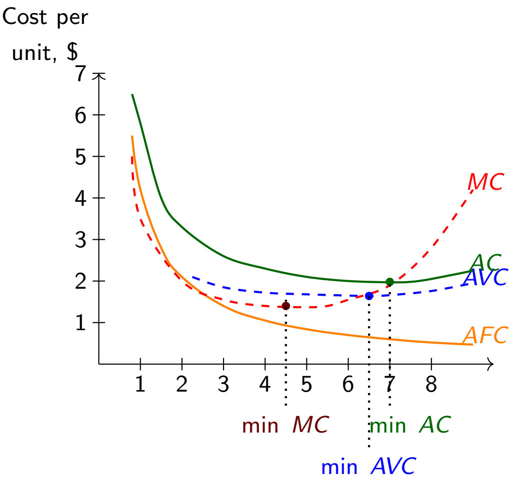

Note: In the first column, the slope of the dashed red and blue lines are the AC and AVC, which correspond to the MC at their minimum average cost.

Costs in the Short Run and Long Run

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  - In the short run, **diminishing marginal returns** to labour determine the shape of the production function.
  - Moreover, labor is the only variable input, so $VC = wL$ (wage $\times$ labour)
  - This means that as output increases, variable costs increase more than proportionally.
  - Diminishing marginal returns mean more labor is needed to produce one more unit of output.

Diminishing Marginal Returns

Diminishing marginal returns determine the shape of the marginal cost curve.

- Recall that $MC = \frac{\Delta VC}{\Delta q}$, and note that with a fixed capital stock, the change in variable cost must be the change in the cost of labor:

$$MC = \frac{\Delta VC}{\Delta q} = w \cdot \frac{\Delta L}{\Delta q}$$

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$\Longrightarrow$ Marginal costs and the marginal product of labor are **inversely related**; MC falls as MPL increases and vice-versa.

Diminishing Marginal Returns

Diminishing marginal returns also determine the shape of the average cost curve.

Recall $AVC = \frac{VC}{q}$, and in the short run labor is the only variable input, so $VC = wL$. Hence:

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Average variable costs and the average product of labor are **inversely related**; AVC falls as AP_L increases and vice versa.

Diminishing Marginal Returns — Summary

To summarize, in the short run:

- The costs associated with fixed inputs are fixed, while the costs from inputs that can be adjusted are variable.
- For a given set of input prices, the shapes of the variable cost curves and the marginal cost curve are determined by the production function.
- Where there are diminishing marginal returns, the variable cost and cost curves become relatively steep as output increases, so AC, AVC, and MC rise with output.
- The AC and AVC curves fall when MC is below them and rise when MC is above them, so MC **cuts both average cost curves at their minimum points**.

Long Run Costs

- In the long run, firms adjust **all** of their inputs so that the cost of production is as low as possible.
- Firms can adjust plant size, design, build new machines, and otherwise adjust inputs that were fixed in the short run.
- Fixed costs are avoidable in the long run – firms can **shut down** operations.
- These costs are no longer sunk, as in the short run.

Long Run Costs

- In the long run, the firm's problem becomes one of **choosing between bundles of inputs**.
- Recall: In the long run, the firm can choose to produce a given quantity of output from many different technically efficient combinations of inputs.
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Answer: the combination that **minimizes costs** for each level of production q

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  – Analogy to consumer theory: need information on preferences and the prices (costs) of goods.

Cost Minimization

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$$C = w \times L + r \times K$$

where w is the wage rate, and r is the rental rate on capital.

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The **isocost line** is all of the combinations of labour and capital that have the **same total cost** $\bar{C}$:

$$\bar{C} = \; w \times L \;+\; r \times K$$

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where w is the wage rate, and r is the rental rate on capital.

We can re-express Axel's isocost line where K is a function of L:

$$K = \frac{\bar{C}}{r} - \frac{w}{r}L$$

Note: the slope of the isocost line is $-w/r$. What are the K and L intercepts?

Isocost Lines

- The slope of the isocost line $-w/r$ tells us at which rate the firm can trade capital for labor on the input market and retain the same total costs.

- If a firm increases its total costs $\bar{C}$, how will the isocost curve shift?

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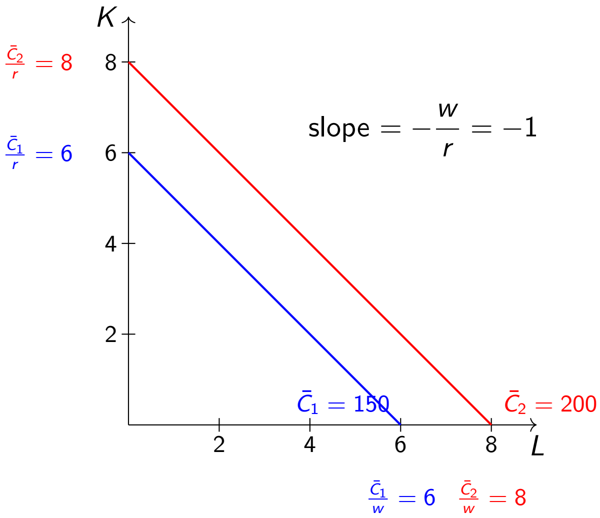
- If a firm increases its total costs $\bar{C}$, how will the isocost curve shift? Parallel shift outward.

Isocost lines, Graphically, for $w = \$25/hr$ and $r = \$25/hr$



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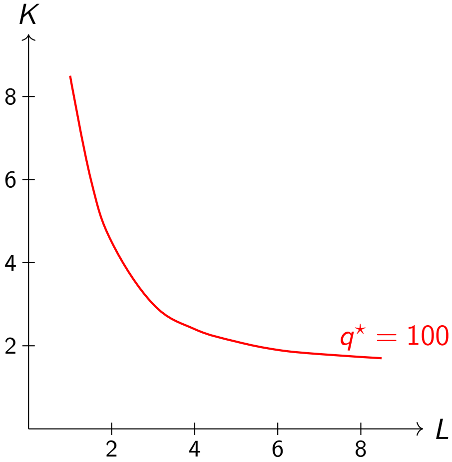
  - The economically efficient outcome occurs when these trade-offs are the **same**.
  - This **minimizes** the cost of production.
  - Occurs when the firm chooses the combination of inputs that is on the **lowest isocost line** that touches the isoquant with the desired level of production.

Cost Minimization: Axel's Coffee Shop

Suppose Axel wants to produce $q^{\star} = 100$ espressos, with $w = \$25/\text{hr}$ and $r = \$25/\text{hr}$.



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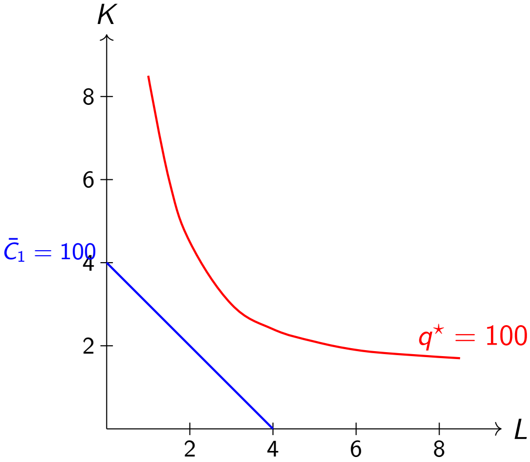

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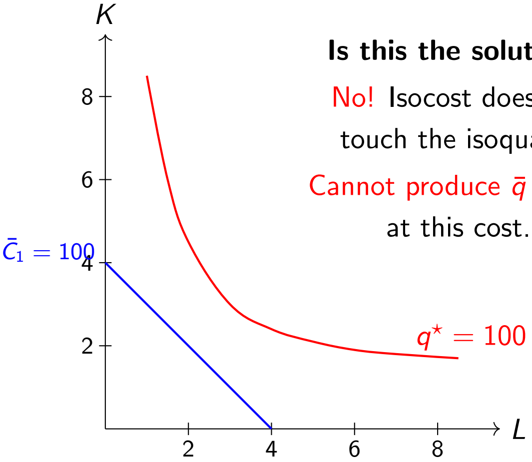

Is this the solution?

No! Isocost does not touch the isoquant.

Cannot produce $\bar{q} = 100$ at this cost.

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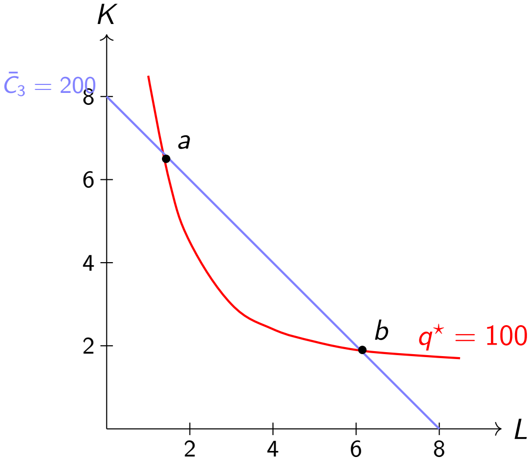

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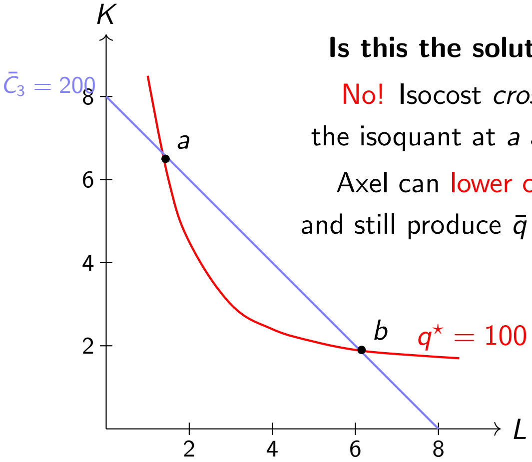

Is this the solution?

No! Isocost *crosses* the isoquant at a and b.

Axel can lower costs and still produce $\bar{q} = 100$.

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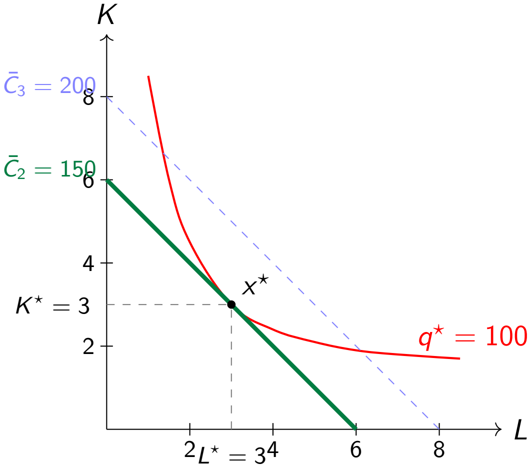
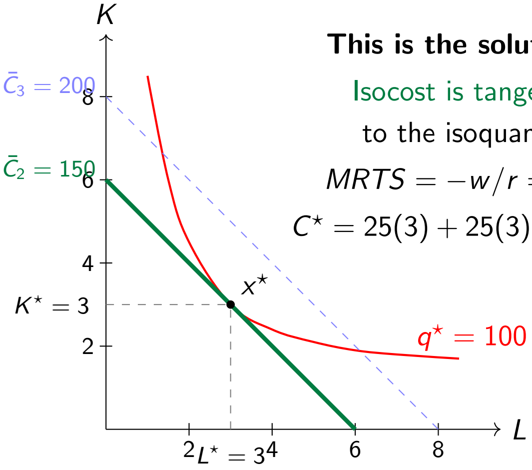


Cost Minimization: Axel's Coffee Shop

Suppose Axel wants to produce $q^\star = 100$ espressos, with $w = \$25/\text{hr}$ and $r = \$25/\text{hr}$.



This is the solution!

Isocost is tangent

to the isoquant.

$$MRTS = -w/r = -1$$

$$C^\star = 25(3) + 25(3) = \$150$$

Cost Minimization — Three Equivalent Rules

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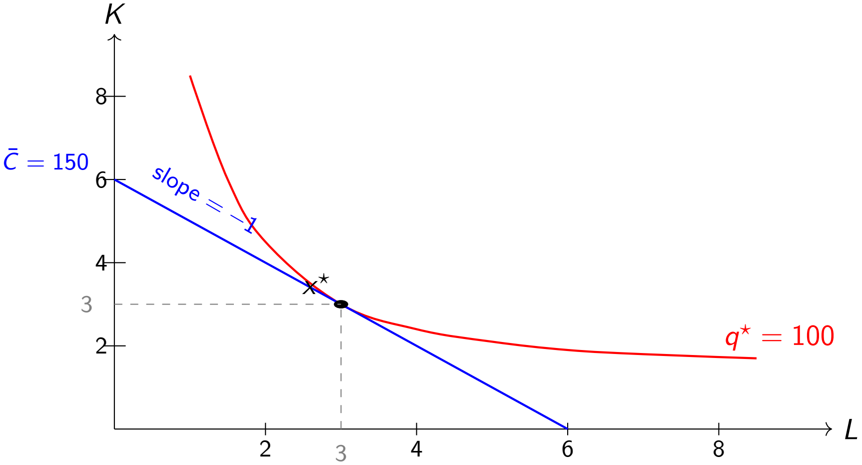
3. **The Last-Dollar Rule**
   $$\frac{MPL}{w} = \frac{MPK}{r}$$
   Cost is minimized if the last dollar spent on labour adds **as much extra output** as the last dollar spent on capital.

Example: Cost Minimization When Input Prices Change (Wage Falls)



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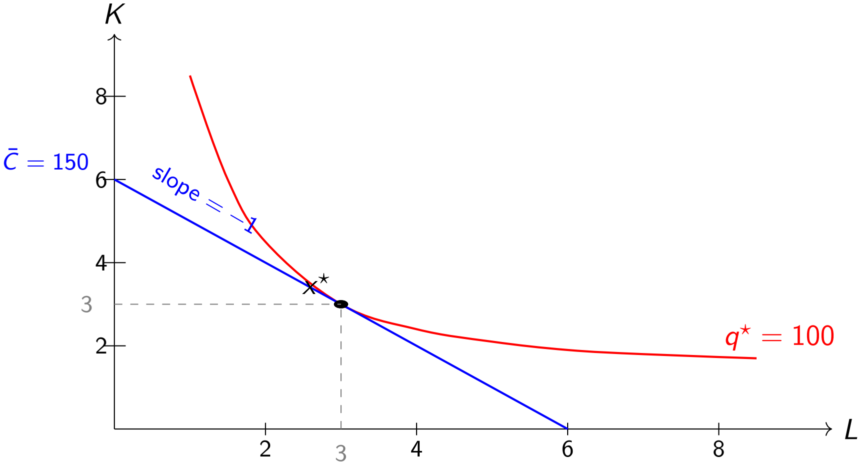
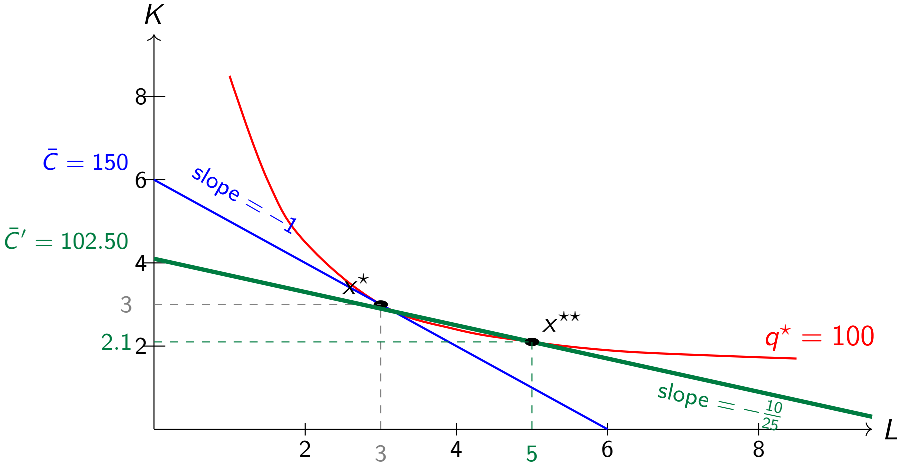


Suppose w falls from \$25 to \$10. Then, isocost **flattens** ($-w/r$ less steep).

Axel substitutes toward more labour and less capital, **and** costs fall.

Example: Cost Minimization When Input Prices Change (Wage Falls)



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The Short and Long Run Average Cost Curve

Long Run Average Costs

- The LRAC curve must be **U-shaped** if a production function has:

  increasing returns to scale at low levels of output,

  constant returns to scale at intermediate levels of output, and

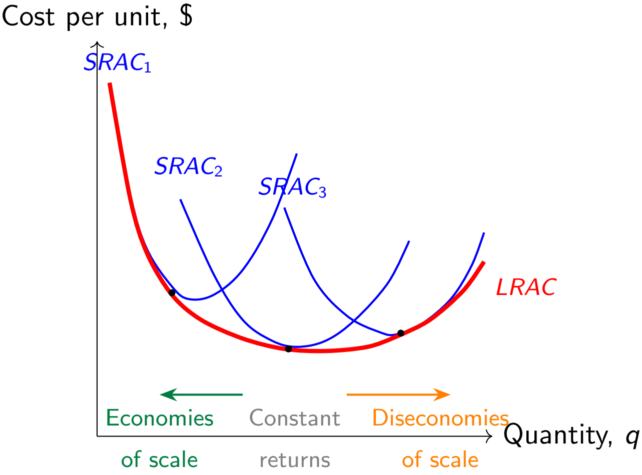
  decreasing returns to scale at high levels of output.

- **Economies of scale** occur if average costs *fall* as output expands.
- **Diseconomies of scale** occur if average costs *rise* as output expands.
- There are **no economies of scale** if average costs do not change with output.

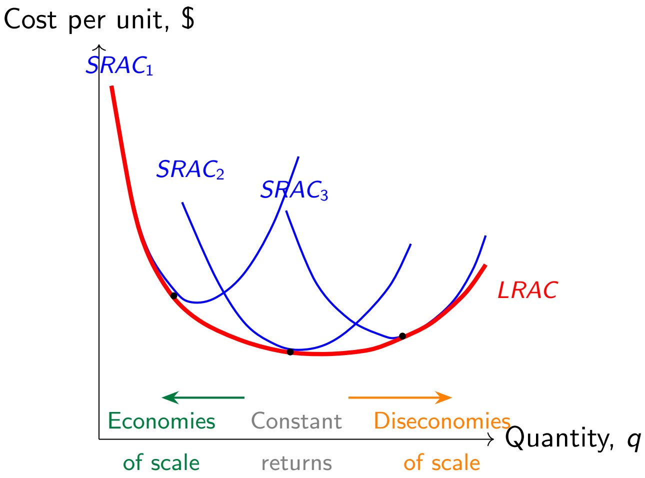
Long Run Average Costs

- It is important to note that the short-run and long-run average cost curves are related.
- Specifically, long-run average costs are always **equal to or below** short-run average costs.
- Why is this?

Long Run Average Costs



Long Run Average Costs



For every output level, long-run costs $\leq$ short-run costs: the firm can always do **at least as well** in the long run by choosing the optimal plant size.

Applications to Management

Automating Labour in Axel's Coffee Shop (In Class)

Axel estimates that his production technology for espresso is

$$q = 10K^{0.5}L^{0.5}$$

in espressos per hour. The current wage for baristas is $\$25/hr$, and the current rental rate on espresso machines is $\$50/hr$.

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A salesman enters the store; he wants to sell a new type of espresso machine that automatically completes orders. 'He says that Axel's new production function could be

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Q: Suppose demand for Axel's coffee at current prices is $q^{\star} = 100$ espressos/hr. Does Axel benefit from adopting this new technology?

Broad Steps to answer this question

- To answer this question, we need to see which technology produces the target output at lower cost.

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- So, we need to solve the cost minimization problem for both production functions.
- In Topic 3A (Consulting Axel), we found the MRTS for a general Cobb-Douglas production function. We can use this formula to find the *MRTS*:

$$\text{Original Technology} \qquad MRTS = \frac{K}{L}$$

$$\text{New Technology} \qquad MRTS = \frac{0.25}{0.75}\frac{K}{L} = \frac{1}{3}\frac{K}{L}$$

Q: New technology lowers the *MRTS* for all K, L. This means labour is worth relatively less than capital now.

Solving the Cost Minimization Problem

- Apply the **tangency rule** ($MRTS = -w/r = -1/2$) for each technology to find the optimal input bundle, then substitute into the production function to find costs at $q^\star = 100$.

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$-K/L = -1/2 \implies K = L/2.$ Substituting into production function:
$100 = 10(L/2)^{0.5}L^{0.5} = 10 \cdot L/\sqrt{2} \implies L^\star = 10\sqrt{2} \approx 14.14,\ K^\star = 5\sqrt{2} \approx 7.07$
$C^\star = 25(14.14) + 50(7.07) \approx \$707/\text{hr}$

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$-\frac{1}{3}\frac{K}{L} = -\frac{1}{2} \implies K = \frac{3}{2}L.$ Substituting into production function:
$100 = 10\left(\frac{3}{2}L\right)^{0.75}L^{0.25} = 10 \cdot \left(\frac{3}{2}\right)^{0.75} \cdot L$
$\implies L^\star = \frac{10}{(3/2)^{0.75}} \approx 7.38, \quad K^\star = \frac{3}{2}(7.38) \approx 11.07$
$C^{\star\star} = 25(7.38) + 50(11.07) \approx \$738/\text{hr}$

Answer

No — Axel should **stick with the original technology**. It produces $q^{\star} = 100$ at \$707/hr vs. \$738/hr under the new technology. The new technology is more capital-intensive, which works *against* Axel when capital ($r = \$50$) is relatively more expensive than labour ($w = \$25$).

Could these new machines be useful?

Q: How *might* your answer change if the cost to rent capital was instead \$10/hr for both the current and new types of espresso machines?

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Sln: Now, Axel could take advantage of a lower cost of capital by adopting a **more capital intensive** technology. That is, Axel could benefit from replacing labour with capital to take advantage of the lower cost of capital.

Producer Theory: Takeaways

- Good management decisions are based on **opportunity cost**.
- In the short run, the relationship between costs and output is determined by fixed costs and diminishing marginal returns.
- Costs are minimized in the long run by using the **last dollar rule**.
- In the long run, the relationship between costs and output is determined by **returns to scale**.

Practice: Chapter 5 and 6 problems (ignore Subsections 6.4/6.5 of text)
Try 3 harder practice problems posted on Canvas, too.

Next: Industrial Organization!

Appendix: Profit Maximization

We learned how a firm can find the cost minimizing input mix to achieve output q.

We can vary q to find the lowest total cost $C(q)$ to produce q units.

In most models we will learn, firms will then choose q to maximize profits, which is just revenue minus costs:

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- In Topic 4, we will study how firms set q in environments where there is 1) a lot of competition, 2) little-to-no competition, and 3) various situations in between.