

Topic 3A: Production Functions

Business Economics, Organization, and Management
(BUEC 211)

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What is a firm? What do firms do? How do we model firms?

- Input decisions
- Production in the Short and Long Run
- Economies of Scale
- Production technology, Innovation (technological change)

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2. 4 workers, each equipped with handheld power saws
3. 2 workers with bench power saws (or table saws)

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1 uses the most labour and the lowest quality capital, while 3 uses the least labour with high quality capital. **Trade off!**

Think of other businesses where the capital-labour mix is important.

The Production Process

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Definition: Production Process

The production process (or technology) of the firm describes how firms transform inputs into outputs.

Inputs are the resources used in production.

- **Capital (K):** Land, buildings, machinery/equipment
- **Labor (L):** Skilled and unskilled (or less-skilled) workers; treated as **differentiated** inputs
- **Materials (M):** Natural resources, raw materials, processed products

Outputs are the result of the production process.

- **Physical product:** Car/computer chip/gasoline
- **Service:** Haircut/consulting services/automobile tune-up

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What are we then assuming about how firms use inputs? Could we relax this assumption?

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K can measure the **amount of capital** (the number of saws) and it can measure the **quality of capital** (e.g. 1 table saw might be equivalent to 10 hand saws).

The Production Function

- Typically, we determine what f is by looking at data on inputs and outputs.
- Empirical evidence suggests that the **Cobb-Douglas** production function fits data for many industries:

$$f(K, L) = A \times L^{\alpha} \times K^{\beta}$$

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- We will see why Cobb-Douglas production fits the data well.

The Production Function

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We will start with **long run** production.

Long Run Production

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We can depict all of the efficient input combinations that produce a given level of output q^* using an **isoquant**.

What are Isoquants?

Definition: Isoquant

Suppose a firm only employs capital and labour and desires to produce q^* units of output. An **isoquant** associated with q^* is the set of all bundles of capital and labour that efficiently produce this level of output:

$$q^* = f(K, L)$$

As an example, consider the case of Axel who is planning on opening a Starbucks franchise.

Both capital and labor are variable; Axel can choose from many different possible combinations of inputs – the number of baristas and the number of different espresso machines – to produce the same level of output.

A Production Spreadsheet

Axel learns that his store will produce espresso (in tens of litres per day) as follows:

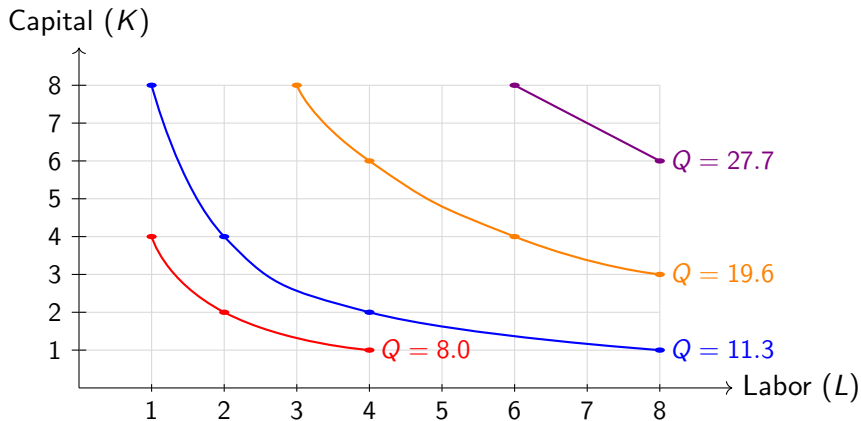
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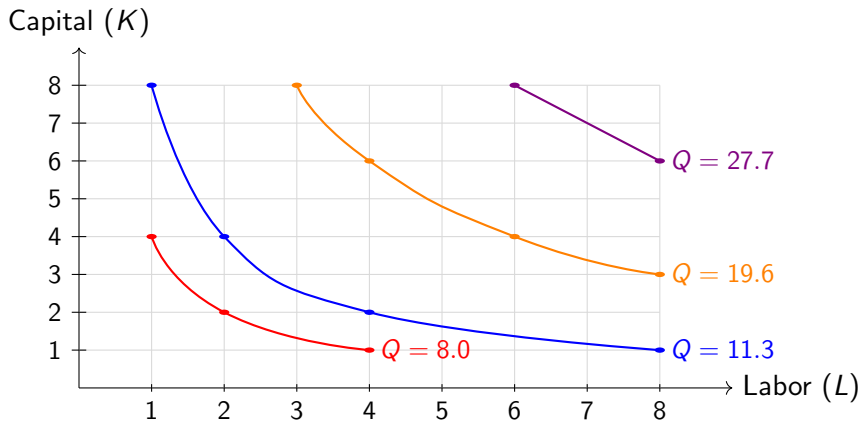
Capital (K)	Labor (L)							
	1	2	3	4	5	6	7	8
8	11.3	16.0	19.6	22.6	25.3	27.7	29.9	32.0
7	10.6	15.0	18.3	21.2	23.7	25.9	28.0	29.9
6	9.8	13.9	17.0	19.6	21.9	24.0	25.9	27.7
5	8.9	12.6	15.5	17.9	20.0	21.9	23.7	25.3
4	8.0	11.3	13.9	16.0	17.9	19.6	21.2	22.6
3	6.9	9.8	12.0	13.9	15.5	17.0	18.3	19.6
2	5.7	8.0	9.8	11.3	12.6	13.9	15.0	16.0
1	4.0	5.7	6.9	8.0	8.9	9.8	10.6	11.3

Each color represents an isoquant (same output level with different K - L combinations)

Isoquants are like Indifference Curves



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Note: Isoquants will continue forever past the end points on the graph, just like with indifference curves.

Isoquants have three key properties:

1. Isoquants farther from the origin represent a higher level of output
2. Isoquants do not cross
3. Isoquants slope downward

The shape of an isoquant is informative for understanding how easily a firm can substitute between inputs to produce a given level of output.

- If inputs are **perfect substitutes**, isoquants are straight lines
- If inputs can not be substituted at all and must be used in **fixed proportions**, isoquants are right angles. (**Perfect Complements!**)
- If inputs can be substituted **imperfectly** for each other, isoquants are **convex** to the origin. (**Just like convex preferences!**)

Perfect Substitutes

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Ex: Electricity from solar panels vs. grid electricity; natural gas vs. oil for heating

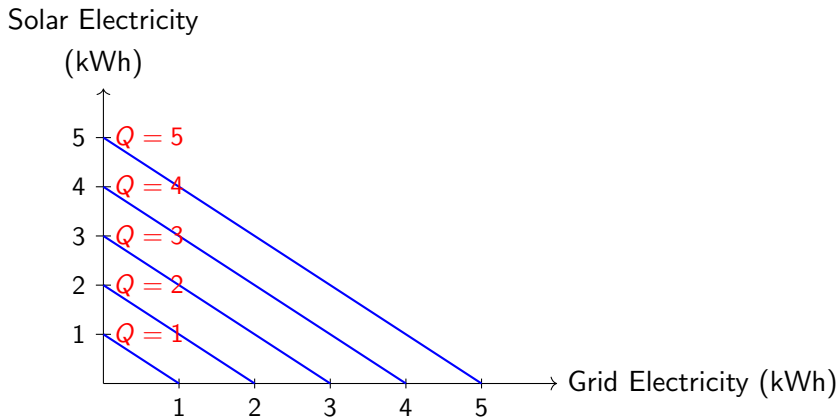
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Ex: Any capital that automates tasks performed by humans (McDonald's serving kiosks)

Isoquants of Perfect Substitutes (Example: Electricity from Different Sources)



Where Q is measured in kWh. For each isoquant, slope = -1 . Inputs (different sources of electricity) can be substituted one-for-one.

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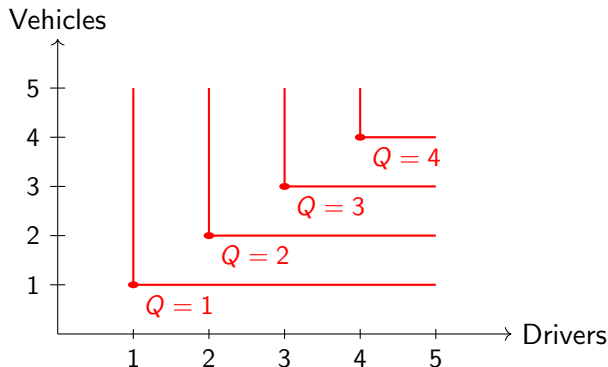
Ex: Drivers and vehicles for ride-sharing platforms or transportation services; pilots and planes for airlines

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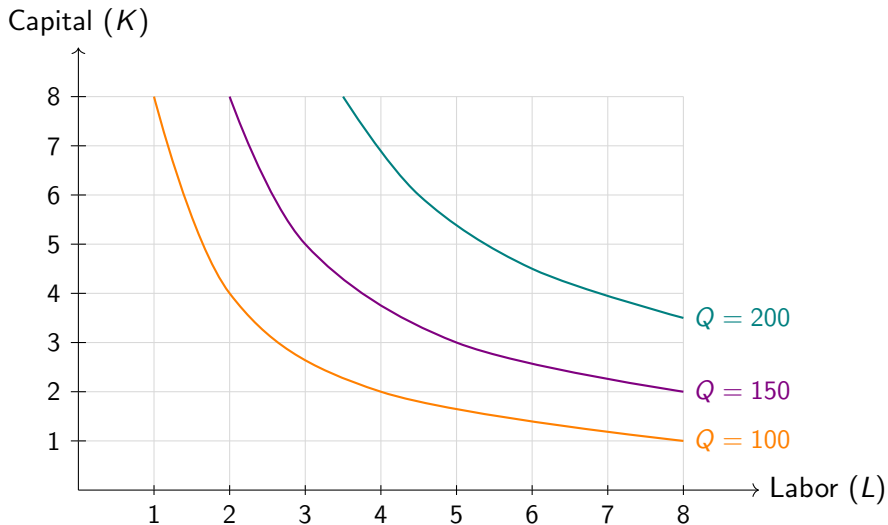
Ex: Many molecular structures (e.g. fertilizer is always some fixed proportion of nitrogen, potassium, and phosphorus);
Computer parts (motherboards, CPU, and a storage drive combine to make a functioning computer)

Isoquants of Perfect Complements



Where Q is measured in trips per hour. Inputs must be used in fixed proportions (1:1 ratio). Extra units of one input without the other produce no additional output.

Typical Isoquants are Between the Extremes



The Marginal Rate of Technical Substitution

The slope of the isoquant is also important.

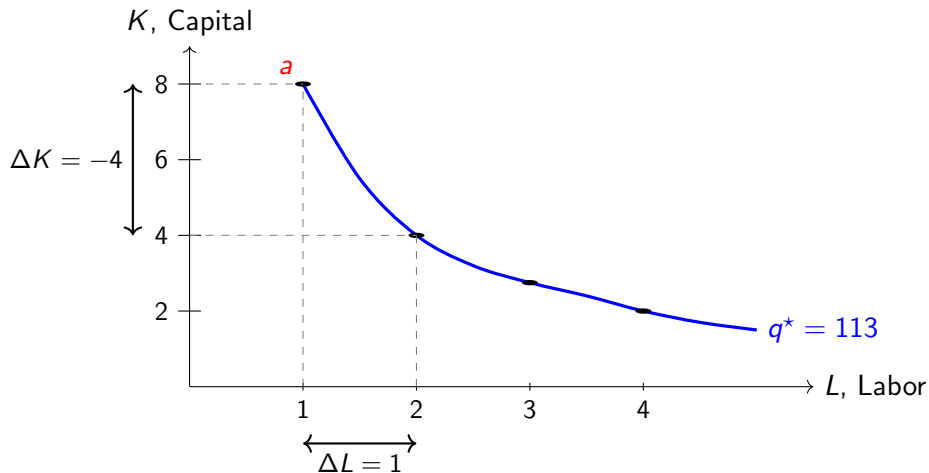
- The slope tells us how costly it is for a firm to replace one input with another, holding output constant at some target q^* .
- The slope of the isoquant is the **Marginal rate of Technical Substitution**

Definition: Marginal Rate of Technical Substitution (MRTS)

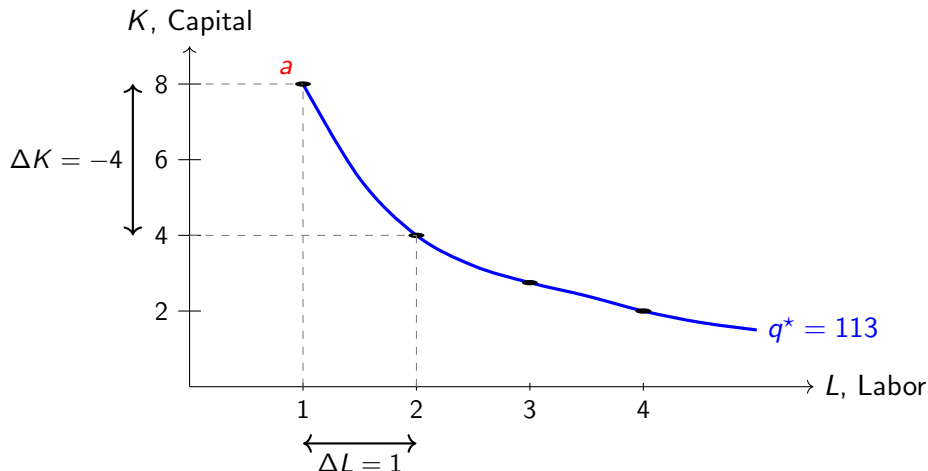
Suppose a firm wants to produce q^* with only K and L used in production. The MRTS defines how much capital must be given up for an additional unit of labour to maintain production q^* :

$$MRTS_{LK} = \frac{\Delta K}{\Delta L}$$

The Marginal Rate of Technical Substitution



The Marginal Rate of Technical Substitution



Suppose Axel wants to produce 113 litres of espresso today. In this case, at the point a , Axel can sell 4 espresso machines and hire 1 worker to maintain the target. $MRTS_{LK} = -4/1 = -4$.

The Marginal Rate of Technical Substitution

Typically, the absolute value of the MRTS is falling along the isoquant.

- Analogous to consumer preferences, this means that inputs are not perfect substitutes in production.
- For most production technologies, using a lot of capital and little labour means an additional unit of labour is worth relatively more than capital. (And vice versa).

The Marginal Product

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- Q: How much does output change if we hire an additional worker? How much does output change if we install one more machine?
- This is crucial for understanding the production function and firm incentives. To answer this question, we need to know the **Marginal Product** of capital and labour:

Definition: Marginal Product (MP)

The marginal product (MP) is the change in total output that results from employing an extra unit of a factor of production, holding other factors constant. For example, the marginal product of labour is defined as:

$$MP_L = \frac{\Delta q}{\Delta L} = \frac{df(K, L)}{dL}$$

Relating MP and MRTS

- We can also relate the MRTS to the marginal productions of labor (MP_L) and capital (MP_K)

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Q: Why is this relation true? Suppose the $MP_L = 2$ and the $MP_K = 1$. Then, giving up 2 units of capital reduces output by $2 \times MP_K = 2$, and hiring one more worker increases output by $1 \times MP_L = 2$. Trading 2 units of capital for one worker keeps output constant (along an isoquant). Thus, $MRTS_{LK} = -(2/1) = -2$.

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The average product of a factor of production is the ratio of total output to the total use of that factor. For labour,

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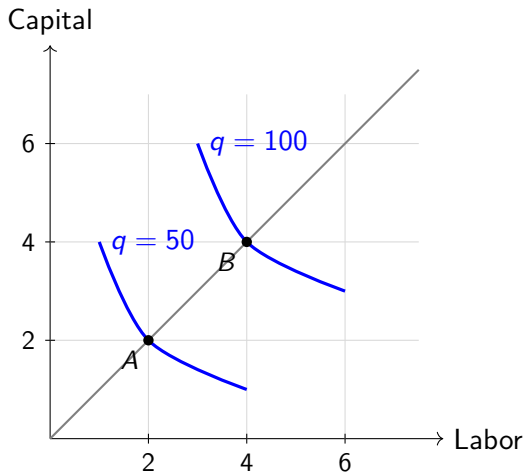
Note: AP_L is generally **not** the MP_L for most production functions. The AP_L is about average productivity of the current workforce, and the MP_L is about the productivity of an additional worker hired.

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- An important related question: How does output change if a firm increases all of its inputs proportionately?
 - The answer to this question requires knowing the **Returns to Scale**.

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- An important related question: How does output change if a firm increases all of its inputs proportionately?
 - The answer to this question requires knowing the **Returns to Scale**. There are 3 categories:
 1. Constant Returns to Scale
 2. Increasing Returns to Scale
 3. Decreasing Returns to Scale

Constant Returns to Scale (CRS)



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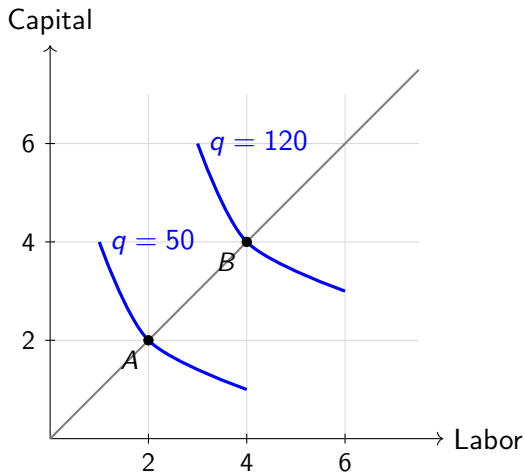
If all inputs are increased by a factor t , output increases by exactly the same factor t . I.e.

$$f(2L, 2K) = 2f(L, K)$$

Example:

- Double all inputs $A \rightarrow B$:
 $(2, 2) \rightarrow (4, 4)$
- Output doubles:
 $50 \rightarrow 100$

Increasing Returns to Scale (IRS)



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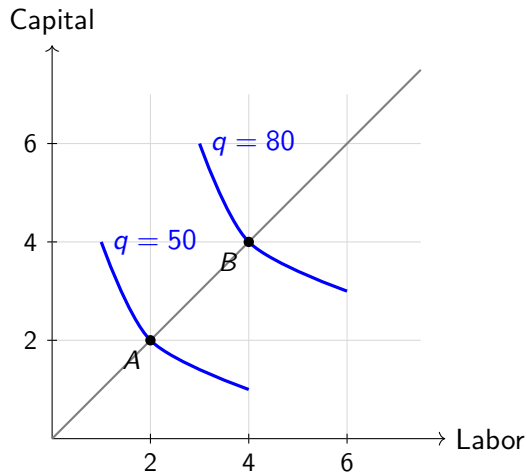
If all inputs are increased by a factor t ,
output increases by a factor greater than t .
I.e.

$$f(2L, 2K) > 2f(L, K)$$

Example:

- Double all inputs $A \rightarrow B$:
 $(2, 2) \rightarrow (4, 4)$
- Output more than doubles:
 $50 \rightarrow 120$

Decreasing Returns to Scale (DRS)



Decreasing Returns to Scale (DRS)

If all inputs are increased by a factor t ,
output increases by less than the factor t .
I.e.

$$f(2L, 2K) < 2f(L, K)$$

Example:

- Double all inputs $A \rightarrow B$:
 $(2, 2) \rightarrow (4, 4)$
- Output less than doubles:
 $50 \rightarrow 80$

Examples:

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2. **Spread of Fixed Costs:** Firms may be more efficient at scale if production requires large fixed costs to build intellectual capital, i.e. purchase software, researching patents (incl. drugs); Use of Information Technology fixed inputs (i.e. payroll software) becomes more efficient at scale, too.

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Empirical evidence suggests most industries exhibit constant or near-constant returns to scale

Manufacturing is slightly increasing returns, services are generally decreasing returns

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We will advise on the 1) productivity of capital and labour, and 2) the returns to scale

Short Run Production

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where $\bar{K}$ is a fixed input (not allowed to be changed).

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In this case, output can only change if the firm changes L .

Short Run Example: Selling Ice Cream

As an example, again consider Axel's coffee shop.

Axel's capital stock is fixed over the next year; he currently has 6 espresso machines and is unable to add more due to space constraints. He needs a plan for the short run before he can scale up his business.

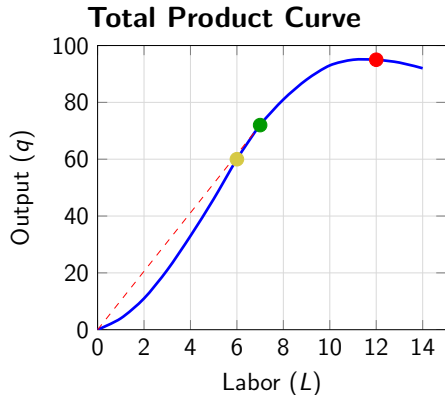
Axel is interested in understanding how changing his workforce affects output and productivity.

Tabulating Axel's Input Use and Productivity

Capital (K)	Labor (L)	Output (q)	MP_L	AP_L
6	0	0	—	—
6	1	4	4	4.0
6	2	11	7	5.5
6	3	21	10	7.0
6	4	33	12	8.3
6	5	46	13	9.2
6	6	60	14	10.0
6	7	72	12	10.3
6	8	81	9	10.1
6	9	88	7	9.8
6	10	93	5	9.3
6	11	95	2	8.6
6	12	95	0	7.9

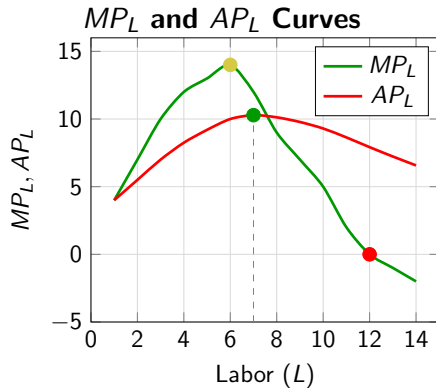
Short Run Production: Axel's Coffee Shop with Fixed Capital ($K = 6$)

Plotting Axel's Input Use and Productivity



TP: Total Product.

Dashed: Slope = AP_L at $L = 7$, which also equals to the MP_L (derivative of TP curve)



$MP_L = AP_L$ where AP_L peaks; $\approx L = 7$

Production in the Short Run

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- If the MP_L curve is above the AP_L curve, the AP_L curve must rise with extra workers
- If the MP_L curve is below the AP_L curve, the AP_L curve must fall with extra workers
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The AP_L and MP_L curves can be derived from the total product curve. For L workers:

- The AP_L is equal to the slope of a straight line from the origin to the total product curve with L workers
- The MP_L is the slope of the total product curve with L workers

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Definition: Law of Diminishing Marginal Returns

If a firm keeps increasing the usage of an input, holding all other inputs and technology constant, then the corresponding increases in output will eventually become smaller (diminish). That is, MP_L is decreasing in L when L is large.

How fast diminishing returns “kick in” depends on the firm’s technology.

Q: What about the Cobb-Douglas production function (see slide 34)?

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If a firm keeps increasing the usage of an input, holding all other inputs and technology constant, then the corresponding increases in output will eventually become smaller (diminish). That is, MP_L is decreasing in L when L is large.

How fast diminishing returns “kick in” depends on the firm’s technology.

Q: What about the Cobb-Douglas production function (see slide 34)? $MP_L/AP_L = \alpha$

Innovation

Recall: A production function represents a method for turning inputs into outputs.

The underlying production process that the production function captures depends on available **technology**:

- This can mean technology in the sense of new ideas, methods, or devices to achieve higher output
- But, it can also mean the quality of firm management or ways of organizing firms that achieve higher output

Changes in technology will affect the production function.

Definition: Process Innovation

A process innovation is a new idea, device or method that allows more output to be produced with the same level of inputs.

Example: Artificial Intelligence

- Firms can scale up production of services originally done solely by humans
- Firms now integrate AI to perform customer service, or to do the job of secretaries and administrative assistants

Definition: Organizational Innovation

An organizational innovation is a new way of organizing a firm that allows more output to be produced with a given level of inputs.

Example: Apple, Henry Ford

- Introduced interchangeable parts
- Introduced a conveyor belt and assembly line into his operations
- Allowed for **specialization** of labour force at different points on the assembly line

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AI (or large language models) can be thought of as labour-saving progress.

Firm Theory Part A: Key Takeaways

- The production function tells us how firms are able to turn inputs into output
- In the short run, firms alter production decisions by changing the use of variable inputs
- In the long run, firms alter production decisions by changing the use of all inputs
- In the long run, the effects of changing input levels depends on returns to scale
- The quantity of output obtained from a level of inputs can be increased by technological progress and through organizational innovation

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Next: Cost Functions and Cost Minimization.

Review: Chapter 5 of the textbook + practice problems