Topic 1C: Elasticity

Business Economics, Organization, and Management
(BUEC 211)

James Macek

Alberta School of Business
University of Alberta

Overview

- Today, we will introduce the concept of an **elasticity**.

- We will use this concept in two managerial applications:

  1. Price effects of a supply and demand change

  2. How much are consumers and producers harmed by a tax?

Elasticity

- Two important questions for managers:

  Suppose there is an increase in demand. By how much do equilibrium prices and output change?

Elasticity

- Two important questions for managers:

  Suppose there is an increase in demand. By how much do equilibrium prices and output change?

  Suppose there is a decrease in supply. By how much do equilibrium prices and output change?

Elasticity

- Two important questions for managers:

  Suppose there is an increase in demand. By how much do equilibrium prices and output change?

  Suppose there is a decrease in supply. By how much do equilibrium prices and output change?

- Answers depend on the **elasticity of supply** and **demand**, respectively.

Demand Elasticity

Elasticity of Demand: Definition

- The elasticity of demand tells us how **responsive** demand is to changes in price.
- Specifically, if prices increase by 1%, how much does quantity demand increase in **percentage terms**?

Elasticity of Demand: Definition

- The elasticity of demand tells us how **responsive** demand is to changes in price.
- Specifically, if prices increase by 1%, how much does quantity demand increase in **percentage terms**?

Elasticity of Demand

Let $D(P)$ be the demand function. The elasticity of demand ϵ_D is

$$\epsilon_D = \underbrace{\frac{\Delta D/D}{\Delta P/P}}_{\text{Percentage change in Demand / Percentage change in Prices}} = \frac{\Delta D}{\Delta P} \times \frac{P}{D}$$

Elasticity of Demand: Definition

- The elasticity of demand tells us how **responsive** demand is to changes in price.
- Specifically, if prices increase by 1%, how much does quantity demand increase in **percentage terms**?

Elasticity of Demand

Let $D(P)$ be the demand function. The elasticity of demand ϵ_D is

$$\epsilon_D = \underbrace{\frac{\Delta D/D}{\Delta P/P}}_{\text{Percentage change in Demand / Percentage change in Prices}} = \frac{\Delta D}{\Delta P} \times \frac{P}{D}$$

Q: Which groups of consumers are more likely to be price sensitive? List them.

Elasticity of Demand: Calculation

Q: Suppose an individual's demand function for gasoline is $D(P) = 50 - 5P$ litres per week. If prices are currently $P = \$4$, how price sensitive is this consumer as measured by elasticity?

Elasticity of Demand: Calculation

Q: Suppose an individual's demand function for gasoline is $D(P) = 50 - 5P$ litres per week. If prices are currently $P = \$4$, how price sensitive is this consumer as measured by elasticity?

- Solution: Using the formula:

$$\epsilon_D = \frac{\Delta D}{\Delta P} \times \frac{P}{D} = -5 \times \frac{P}{50 - 5P}$$

Elasticity of Demand: Calculation

Q: Suppose an individual's demand function for gasoline is $D(P) = 50 - 5P$ litres per week. If prices are currently $P = \$4$, how price sensitive is this consumer as measured by elasticity?

- Solution: Using the formula:

$$\epsilon_D = \frac{\Delta D}{\Delta P} \times \frac{P}{D} = -5 \times \frac{P}{50 - 5P}$$

- Substitute $P = 4$,

$$\epsilon_D = -5 \times \frac{4}{50 - 5 \times 4} = -\frac{20}{30} = -\frac{2}{3}$$

Elasticity of Demand: Calculation

Q: Suppose an individual's demand function for gasoline is $D(P) = 50 - 5P$ litres per week. If prices are currently $P = \$4$, how price sensitive is this consumer as measured by elasticity?

- Solution: Using the formula:

$$\epsilon_D = \frac{\Delta D}{\Delta P} \times \frac{P}{D} = -5 \times \frac{P}{50 - 5P}$$

- Substitute $P = 4$,

$$\epsilon_D = -5 \times \frac{4}{50 - 5 \times 4} = -\frac{20}{30} = -\frac{2}{3}$$

- Interpretation: a 1% increase in prices leads to demand decreasing by 0.66%. We say demand is **inelastic** at this price because $|\epsilon_D| < 1$.

Elasticity

- Some key concepts:

  1. Inelastic demand means an absolute value of the elasticity is less than one (i.e. the relative magnitude of the quantity response is smaller than the price change).
  2. Elastic demand means an absolute value elasticity larger than one.
  3. Perfectly inelastic demand has an elasticity of zero
  4. Perfectly elastic demand has an infinite elasticity (in absolute value).
  5. Demand is neither inelastic or elastic when the absolute value of the elasticity is one.

Elasticity

- Some key concepts:
    1. Inelastic demand means an absolute value of the elasticity is less than one (i.e. the relative magnitude of the quantity response is smaller than the price change).
    2. Elastic demand means an absolute value elasticity larger than one.
    3. Perfectly inelastic demand has an elasticity of zero
    4. Perfectly elastic demand has an infinite elasticity (in absolute value).
    5. Demand is neither inelastic or elastic when the absolute value of the elasticity is one.
- Simple way to categorize demand responses of consumers to either low price sensitivity (inelastic) or high price sensitivity (elastic)

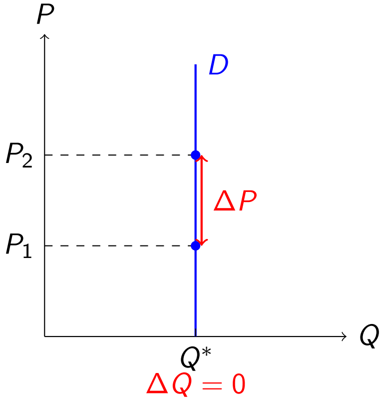
Perfectly Inelastic Demand

Characteristics:

- Elasticity: $\epsilon_D = 0$
- Vertical inverse demand curve
- Demand unresponsive to price
- Consumers will purchase the good regardless of price

Extreme Examples:

- Life-saving medication
- Insulin for diabetics

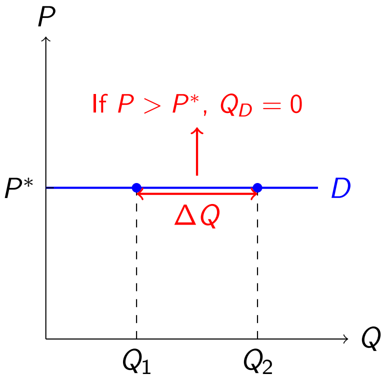


Interpretation: Price increases from P_1 to P_2, but quantity remains at Q^*. Consumers have no ability or willingness to reduce consumption.

Perfectly Elastic Demand

Characteristics:

- Elasticity: $\epsilon_D = -\infty$
- Horizontal inverse demand curve
- Any price increase causes demand to fall to zero



Examples:

- No true example
- Demand becomes more elastic if there are other **substitute goods**

Interpretation: At price P^*, consumers will buy any quantity. If price rises above P^*, demand drops to zero (consumers switch to perfect substitutes).

Demand Elasticity and Marginal Revenue

- If Demand is $D(P)$, then total expenditure on the good as a function of price is

$$R = D(P) \times P$$

Demand Elasticity and Marginal Revenue

- If Demand is $D(P)$, then total expenditure on the good as a function of price is

$$R = D(P) \times P$$

  This is also **total revenues** received by all firms in the market.

Demand Elasticity and Marginal Revenue

- If Demand is $D(P)$, then total expenditure on the good as a function of price is

$$R = D(P) \times P$$

  This is also **total revenues** received by all firms in the market.

- Then, the **marginal revenue** of a small price change is

$$\frac{dR}{dP} = \underbrace{\frac{dD}{dP} P + D(P)}_{\text{Product rule from calculus}}$$

Demand Elasticity and Marginal Revenue

- If Demand is $D(P)$, then total expenditure on the good as a function of price is

$$R = D(P) \times P$$

  This is also **total revenues** received by all firms in the market.

- Then, the **marginal revenue** of a small price change is

$$\frac{dR}{dP} = \underbrace{\frac{dD}{dP} P + D(P)}_{\text{Product rule from calculus}} = D(P)\left[1 + \frac{dD}{dP}\frac{P}{D}\right]$$

Demand Elasticity and Marginal Revenue

- If Demand is $D(P)$, then total expenditure on the good as a function of price is

$$R = D(P) \times P$$

  This is also **total revenues** received by all firms in the market.

- Then, the **marginal revenue** of a small price change is

$$\frac{dR}{dP} = \underbrace{\frac{dD}{dP}P + D(P)}_{\text{Product rule from calculus}} = D(P)\left[1 + \frac{dD}{dP}\frac{P}{D}\right]$$

$$= D(P) \times [1 + \epsilon_D]$$

Demand Elasticity and Marginal Revenue: Interpretation

$$\frac{dR}{dP} = D(P) \times [1 + \epsilon_D]$$

Demand Elasticity and Marginal Revenue: Interpretation

$$\frac{dR}{dP} = D(P) \times [1 + \epsilon_D]$$

Another interpretation of ϵ_D:

1. If $\epsilon_D < -1$ (elastic), then marginal revenue is negative; firms would find **increased revenues** if **prices decreased**

Demand Elasticity and Marginal Revenue: Interpretation

$$\frac{dR}{dP} = D(P) \times [1 + \epsilon_D]$$

Another interpretation of ϵ_D:

1. If $\epsilon_D < -1$ (elastic), then marginal revenue is negative; firms would find **increased revenues** if **prices decreased**
2. If $\epsilon_D > -1$ (inelastic), then marginal revenue is positive; firms would find **increased revenues** if **prices increased**

Demand Elasticity and Marginal Revenue: Interpretation

$$\frac{dR}{dP} = D(P) \times [1 + \epsilon_D]$$

Another interpretation of ϵ_D:

1. If $\epsilon_D < -1$ (elastic), then marginal revenue is negative; firms would find **increased revenues** if **prices decreased**
2. If $\epsilon_D > -1$ (inelastic), then marginal revenue is positive; firms would find **increased revenues** if **prices increased**
3. If $\epsilon_D = -1$ (neither), then firms would earn **same revenue** if prices changed

Example: Price effect of a supply decrease

Q: Suppose the supply of oil is initially fixed at $S(P) = 25$ litres per week. Then, a world shortage of oil changes the supply curve to $S(P) = 20$ litres per week. If the demand curve is $D(P) = 50 - 5P$, how do prices and quantities change? How does this (approximately) depend on the elasticity of demand?

Example: Price effect of a supply decrease

Q: Suppose the supply of oil is initially fixed at $S(P) = 25$ litres per week. Then, a world shortage of oil changes the supply curve to $S(P) = 20$ litres per week. If the demand curve is $D(P) = 50 - 5P$, how do prices and quantities change? How does this (approximately) depend on the elasticity of demand?

- **Direct Solution**: We know that equilibrium quantities change from 25 to 20, representing a percentage change of $\frac{20-25}{25} = -\frac{1}{5} = -20\%$.

Example: Price effect of a supply decrease

Q: Suppose the supply of oil is initially fixed at $S(P) = 25$ litres per week. Then, a world shortage of oil changes the supply curve to $S(P) = 20$ litres per week. If the demand curve is $D(P) = 50 - 5P$, how do prices and quantities change? How does this (approximately) depend on the elasticity of demand?

- **Direct Solution**: We know that equilibrium quantities change from 25 to 20, representing a percentage change of $\frac{20-25}{25} = -\frac{1}{5} = -20\%$.
- The price of the initial equilibrium must solve

$$50 - 5P = 25 \quad \Longrightarrow \quad P_0^\star = \frac{25}{5} = \$5 \tag{1}$$

- Exercise: show that the equilibrium price after the supply change is $P_1^\star = \$6$, meaning that prices rise by $100 \times \frac{6-5}{5} = 20\%$!

Example: price effect of a supply decrease

- A **different solution** uses the price elasticity of demand at the initial equilibrium to approximate how prices changed.

Example: price effect of a supply decrease

- A **different solution** uses the price elasticity of demand at the initial equilibrium to approximate how prices changed.
- The elasticity of demand at the initial equilibrium is

$$\epsilon_D = -5 \times \frac{5}{25} = -1$$

Example: price effect of a supply decrease

- A **different solution** uses the price elasticity of demand at the initial equilibrium to approximate how prices changed.
- The elasticity of demand at the initial equilibrium is

$$\epsilon_D = -5 \times \frac{5}{25} = -1$$

Example: price effect of a supply decrease

- A **different solution** uses the price elasticity of demand at the initial equilibrium to approximate how prices changed.
- The elasticity of demand at the initial equilibrium is

$$\epsilon_D = -5 \times \frac{5}{25} = -1$$

- Since we know equilibrium quantities fall by 20%, we can approximate the price change with the formula

$$\underbrace{\%\Delta P}_{\text{\% Change in prices}} = \underbrace{\frac{\Delta P/P}{\Delta Q/Q}}_{\text{Inverse elasticity of demand } 1/\epsilon_D} \times \underbrace{\%\Delta Q}_{\text{-20\%}}$$

Example: price effect of a supply decrease

- A **different solution** uses the price elasticity of demand at the initial equilibrium to approximate how prices changed.
- The elasticity of demand at the initial equilibrium is

$$\epsilon_D = -5 \times \frac{5}{25} = -1$$

- Since we know equilibrium quantities fall by 20%, we can approximate the price change with the formula

$$\underbrace{\%\Delta P}_{\text{\% Change in prices}} = \underbrace{\frac{\Delta P/P}{\Delta Q/Q}}_{\text{Inverse elasticity of demand } 1/\epsilon_D} \times \underbrace{\%\Delta Q}_{\text{-20\%}}$$

So, we know that prices increased by approximately 20% because the elasticity of demand is one.

Supply Elasticity

Elasticity of Supply: Definition

- The elasticity of supply tells us how **responsive** supply is to changes in price.
- Specifically, if prices increase by 1%, how much does quantity supplied increase in **percentage terms**?

Elasticity of Supply: Definition

- The elasticity of supply tells us how **responsive** supply is to changes in price.
- Specifically, if prices increase by 1%, how much does quantity supplied increase in **percentage terms**?

Elasticity of Supply

Let $S(P)$ be the supply function. The elasticity of supply ϵ_S is

$$\epsilon_S = \underbrace{\frac{\Delta S/S}{\Delta P/P}}_{\text{Percentage change in Supply/ Percentage change in Prices}} = \frac{\Delta S}{\Delta P} \times \frac{P}{S}$$

Elasticity of Supply: Definition

- The elasticity of supply tells us how **responsive** supply is to changes in price.
- Specifically, if prices increase by 1%, how much does quantity supplied increase in **percentage terms**?

Elasticity of Supply

Let $S(P)$ be the supply function. The elasticity of supply ϵ_S is

$$\epsilon_S = \underbrace{\frac{\Delta S/S}{\Delta P/P}}_{\text{Percentage change in Supply/ Percentage change in Prices}} = \frac{\Delta S}{\Delta P} \times \frac{P}{S}$$

Q: Which products do you think are produced at a high price elasticity? List them.

Elasticity of Supply: Example

Q: Suppose the supply curve for gasoline is $S(P) = 5P$ litres per week. Use the elasticity of supply to predict how gasoline production will proportionally change if rising demand causes prices to change from \$4 to \$5 (a 25% increase).

Elasticity of Supply: Example

Q: Suppose the supply curve for gasoline is $S(P) = 5P$ litres per week. Use the elasticity of supply to predict how gasoline production will proportionally change if rising demand causes prices to change from \$4 to \$5 (a 25% increase).

- Solution: Try this yourself!

    1. Show that the elasticity of supply is one and constant for all price levels.

        You can either use the direct calculation on Slide 11 or logathrimic differentiation (see math notes)

Elasticity of Supply: Example

Q: Suppose the supply curve for gasoline is $S(P) = 5P$ litres per week. Use the elasticity of supply to predict how gasoline production will proportionally change if rising demand causes prices to change from \$4 to \$5 (a 25% increase).

- Solution: Try this yourself!
  1. Show that the elasticity of supply is one and constant for all price levels.

     You can either use the direct calculation on Slide 11 or logathrimic differentiation (see math notes)

  2. What does that imply if prices increase by 25%?

Application: Who pays for a tax?

Tax Burden

- **Key Question:** When the government imposes a tax on a good, who bears the burden?

  The legal incidence (who writes the check to the government)

  The economic incidence (who actually bears the cost in market equilibrium)

  Answer: The economic incidence depends on the *relative elasticities* of supply and demand, **not who cuts the check**.

Example with equal tax burden

- Consider the market for retail gasoline:

Example with equal tax burden

- Consider the market for retail gasoline:

  Demand: $D(P) = 10 - P$

  Supply: $S(P) = P$

Example with equal tax burden

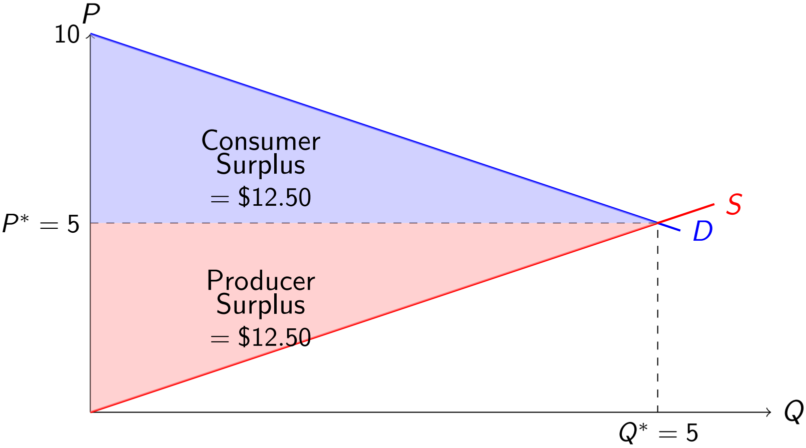
- Consider the market for retail gasoline:

  Demand: $D(P) = 10 - P$

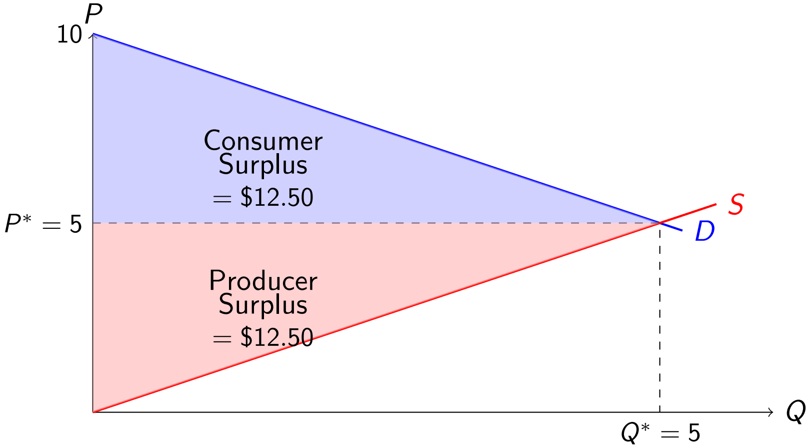
  Supply: $S(P) = P$

  Consider the effects of imposing a \$2 per litre tax paid by gas stations.

Market Equilibrium without the Tax

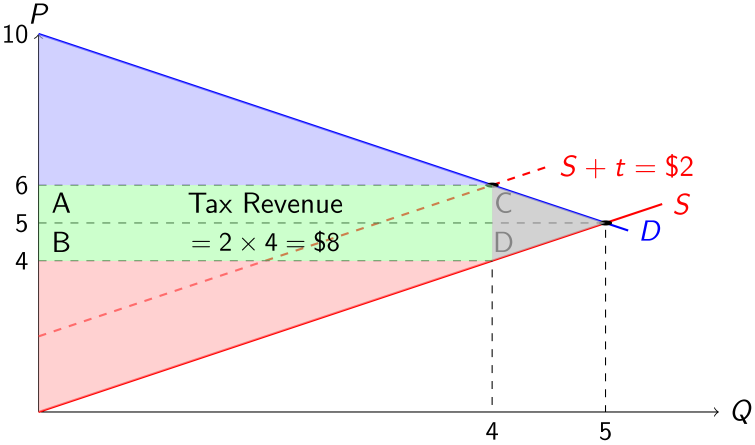


Market Equilibrium without the Tax

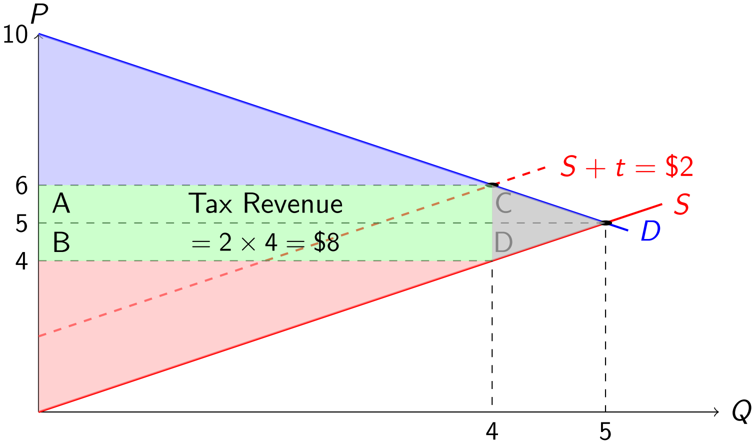


Recall consumer and producer surplus from ECO101 defined by the area of the blue and red triangles, respectively. **Exercise:** show the surplus is \$12.50 by calculating the area.

Welfare Analysis after imposing the tax



Welfare Analysis after imposing the tax



After tax, consumer surplus decreases by area of A + C and producer surplus decreases by B + D. Tax revenue is the area of squares A + B. **Deadweight loss** = C + D triangles.

Welfare Analysis after imposing the tax

- Consumer and producer surplus decreases by the same amount because the elasticity of demand and supply is **equal** at the market equilibrium.

Welfare Analysis after imposing the tax

- Consumer and producer surplus decreases by the same amount because the elasticity of demand and supply is **equal** at the market equilibrium.

Q: What happens when demand is **less elastic** than supply?

This is the likely reality in the market for retail gasoline.

Welfare Analysis after imposing the tax

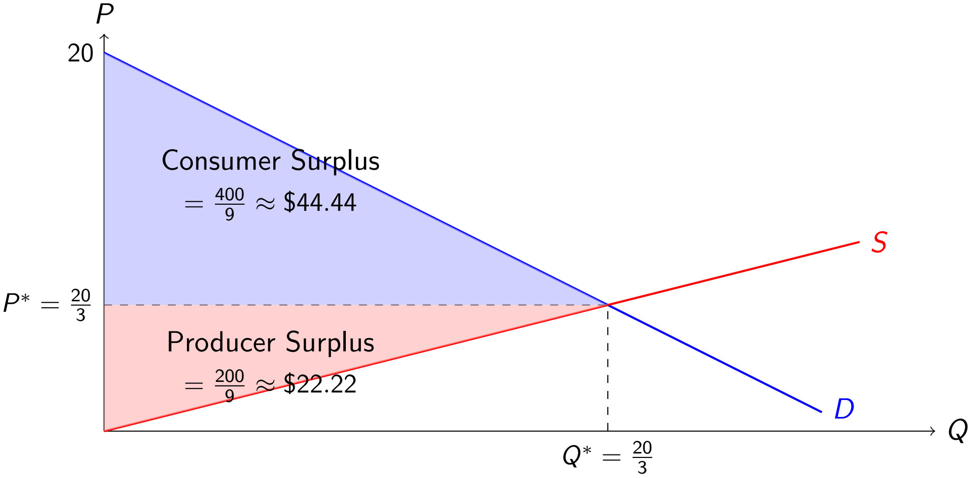
- Consumer and producer surplus decreases by the same amount because the elasticity of demand and supply is **equal** at the market equilibrium.

Q: What happens when demand is **less elastic** than supply?

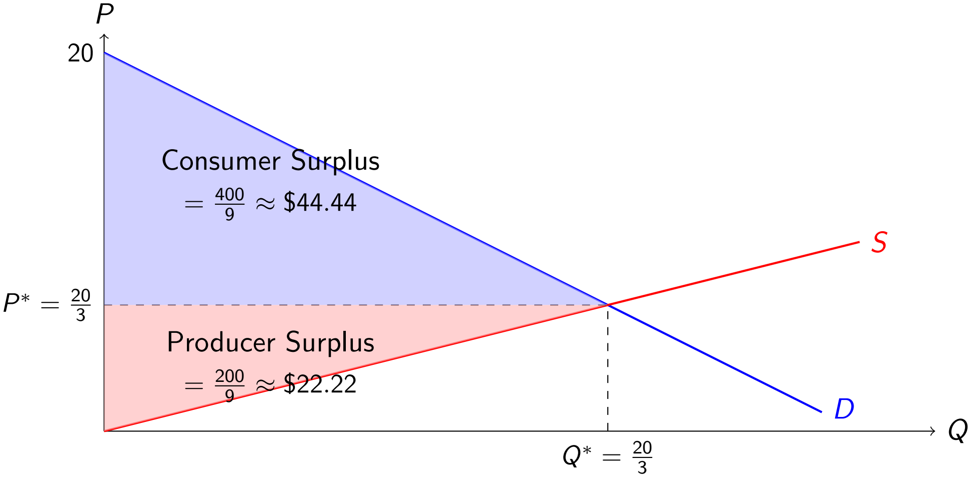
This is the likely reality in the market for retail gasoline.

- Consider a new demand curve $D(P) = 10 - \frac{1}{2}P$
(smaller slope in prices $\implies$ lower elasticity of demand in absolute value)

Market Equilibrium without a tax

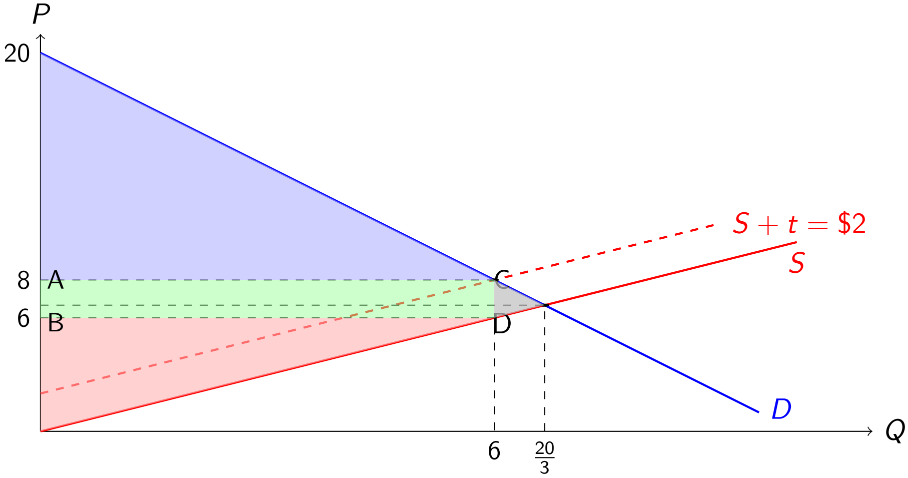


Market Equilibrium without a tax

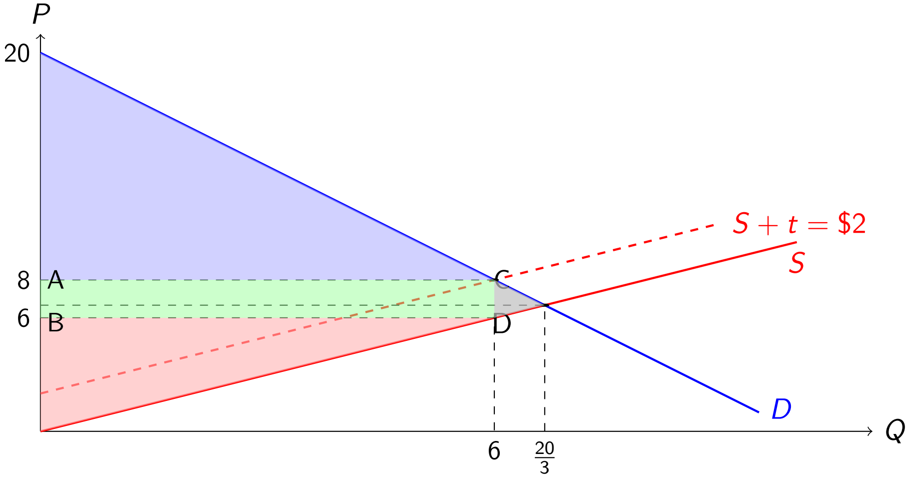


Exercise: show the surplus values by calculating the areas.

Welfare Analysis after imposing the tax



Welfare Analysis after imposing the tax



After tax, consumer surplus decreases by area of A + C and producer surplus decreases by B + D. Tax revenue is the area of squares A + B. **Deadweight loss** = C + D.

Understanding Tax Incidence

- Notice how the loss in consumer surplus is relatively larger than the producer loss when consumers have **inelastic demand**.
- This means consumers face a relatively higher **tax burden** when their demand is inelastic.

Understanding Tax Incidence

- Notice how the loss in consumer surplus is relatively larger than the producer loss when consumers have **inelastic demand**.
- This means consumers face a relatively higher **tax burden** when their demand is inelastic.
- General formula for the formula sheet in tests:

$$\text{Consumer Tax Burden Share} \approx \frac{\epsilon_S}{\epsilon_S - \epsilon_D}$$

Understanding Tax Incidence

- Notice how the loss in consumer surplus is relatively larger than the producer loss when consumers have **inelastic demand**.

- This means consumers face a relatively higher **tax burden** when their demand is inelastic.

- General formula for the formula sheet in tests:

$$\text{Consumer Tax Burden Share} \approx \frac{\epsilon_S}{\epsilon_S - \epsilon_D}$$

$\Longrightarrow$ The higher the elasticity of supply and the lower the elasticity of demand in absolute value, the higher the tax burden is on consumers.

Ending Off

Other Types of Elasticities (you will need these for practice problems)

- The concept of elasticity applies well beyond prices. Examples:

Other Types of Elasticities (you will need these for practice problems)

- The concept of elasticity applies well beyond prices. Examples:
  1. **Income Elasticity of Demand:**

Other Types of Elasticities (you will need these for practice problems)

- The concept of elasticity applies well beyond prices. Examples:
  1. **Income Elasticity of Demand:**

$$\epsilon_D^Y = \underbrace{\frac{\Delta D}{\Delta Y} \times \frac{Y}{D}}_{\text{Percentage change in Demand/ Percentage change in Income}}$$

Other Types of Elasticities (you will need these for practice problems)

- The concept of elasticity applies well beyond prices. Examples:

  1. **Income Elasticity of Demand:**

$$\epsilon_D^Y = \underbrace{\frac{\Delta D}{\Delta Y} \times \frac{Y}{D}}_{\text{Percentage change in Demand/ Percentage change in Income}}$$

  We say that demand is income inelastic if $\epsilon_D^Y < 1$ (elastic if > 1). Examples: gasoline and housing demand is income inelastic.

Other Types of Elasticities (you will need these for practice problems)

- The concept of elasticity applies well beyond prices. Examples:

  1. **Income Elasticity of Demand:**

  $$\epsilon_D^Y = \underbrace{\frac{\Delta D}{\Delta Y} \times \frac{Y}{D}}_{\text{Percentage change in Demand/ Percentage change in Income}}$$

  We say that demand is income inelastic if $\epsilon_D^Y < 1$ (elastic if > 1). Examples: gasoline and housing demand is income inelastic.

  2. **Cross-Price Elasticity of Demand:**
  Let P_c be the price of a different good (i.e. cars). Then, the elasticity of gas demand with respect to car price is:

  $$\epsilon_D^c = \underbrace{\frac{\Delta D}{\Delta P_c} \times \frac{P_c}{D}}_{\text{Percentage change in Gas Demand/ Percentage change in Price of Another Good (Cars)}}$$

Other Types of Elasticities (you will need these for practice problems)

- The concept of elasticity applies well beyond prices. Examples:

  1. **Income Elasticity of Demand:**

     $$\epsilon_D^Y = \underbrace{\frac{\Delta D}{\Delta Y} \times \frac{Y}{D}}_{\text{Percentage change in Demand/ Percentage change in Income}}$$

     We say that demand is income inelastic if $\epsilon_D^Y < 1$ (elastic if > 1). Examples: gasoline and housing demand is income inelastic.

  2. **Cross-Price Elasticity of Demand:**
     Let P_c be the price of a different good (i.e. cars). Then, the elasticity of gas demand with respect to car price is:

     $$\epsilon_D^c = \underbrace{\frac{\Delta D}{\Delta P_c} \times \frac{P_c}{D}}_{\text{Percentage change in Gas Demand/ Percentage change in Price of Another Good (Cars)}}$$

What's Next?

- Next topic: understanding how we derive demand functions (and demand elasticity) from a model of **consumer behaviour**.

    1. Introduce the concept of utility functions
    2. Utility maximization subject to budget constraints
    3. Complement goods and substitutes

What's Next?

- Next topic: understanding how we derive demand functions (and demand elasticity) from a model of **consumer behaviour**.
  1. Introduce the concept of utility functions
  2. Utility maximization subject to budget constraints
  3. Complement goods and substitutes
- Do practice problems for Chapter 3.1: both posted practice and practice from the textbook (screenshotted and posted).